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**Introduction**

Logic lies at the heart of mathematical and scientific thinking, and is fundamentally linked to certain elements of language. According to (Ferrari & Gerla, 2015) the constant attention paid to mathematical language, to the distinction between language and metalanguage, and to the notion of interpretation when working with logic makes it a tool suitable for teaching and learning at every educational stage. However, as pointed out in (Durand-Guerrier et al., 2012) the educational role of logic is not always recognised. There may be several reasons for this: on the one hand, formal logic can be seen as an unnecessary tool that risks complicating teaching practice; on the other, some believe that basic logical abilities are developed irrespective of a targeted theoretical treatment. For example, the concept of ‘not’ is <<considered as a very simple notion […] that does not need to be taught or discussed at this [primary school] level>> (Durand-Guerrier, 2021). (Un’altra motivazione è l’insegnamento della matematica funzionale allo sviluppo di skill utili ai futuri gradi e obiettivi scolastici). According to the Author, the lack of an explicit treatment creates difficulties in understanding negation that persist until university level, such as the fact that the connection between the negation of a universal statement and the role of counterexamples is never fully clarified.

The mistrust towards logic in majorly underlined when dealing with symbolism. Citando ancora Durand-Guerier (2021), il fatto di trascurare gli aspetti relativi alla logica e al linguaggio leads to the paradox that mathematical formalism—which should serve to clarify concepts—becomes instead an obstacle to students’ learning (nelle fasi piu avanzate dell’educazione). Indeed, logical formalism is often seen only when it is necessary to express a mathematical concept not related to logic, considerate più nel suo ruolo di abbreviazione sintattica che di chiarificatore semantico. This is illustrated by the fact that a student may encounter quantifiers for the first time in the limit formula—when it is no longer possible to express the concept in words—which features three alternate quantifiers as well as an implication. Introducing the formal symbols for quantifiers this late in the game feels like a missed opportunity, akin to introducing the equality symbol for the first time when dealing with equations. Introducing a symbol to denote a concept both assumes and requires a societal agreement on its meaning, and allows us to become aware that symbols depend on the context of use (Ferrari, 2002).

For instance, the logical conjunction ‘and’ will not capture every ‘and’ used in natural language; however, knowing how to recognise the differences and the similarities in each case and each context is an excellent starting point for learning. (Coppola et al., 2019) provide an interesting analysis of the relationship between language, as an object to manipulate and reflect upon, and the development of logical abilities, considering specific scenarios of social interaction among primary school children (8–9 years old). A child is asked to behave like a robot that only obeys certain commands; in this way, the game encourages the children to construct a simple symbolic language (whose symbols do not correspond to those of standard logic, but the key point is to view logic as an <<explicit expression of some aspects regarding language>> (Coppola et al., 2019) in which each symbol represents an instruction for the robot. Moreover, the children can discover rules to "manipulate” the symbols of the created language (for example, rules that allow them to establish whether two different sequences of symbols can be considered equivalent in some way).

In questo spirito, presentiamo un’esperienza di insegnamento della logica nella scuola primaria. Indeed, we believe that logic, also in its basic formal form (migliorari inglese, simbolismo), supports the development of rational thought and that it is therefore appropriate to dedicate time and space to logic and its symbols from primary education onwards.
L’obiettivo della nostra esperienza non è aggiungere al bagaglio culturale dello studento nozioni di logica, che se non solo se non riprese tendono a scomparire ma rischirebfbero di inficiare l’affect e l’idea della matematica essendo super tecniche, condividiamo quindi in parte le critiche mosse a certi modi di insegnare la logica evidenziate da durand in precendza, ma impare a ragionare in maniera critica sul linguaggio fine a se stesso. Per fare questo quindi super inquired base.

In questa esperienza verranno introdotti simboli formali per identificare concetti, come simboli per predicati e il simbolo delle negazione. Indeed, just as the equality symbol favours in itself the development of language and algorithmic thought, we consider (per esempio) the negation symbol to aid the development of language and rational thought. Equality and addition are similarly simple notions, and yet the necessity of having formal symbols to represent these concepts from the first years of primary education is universally recognised. The idea that an understanding of basic logical concepts can be acquired automatically through standard mathematical teaching is wishful thinking.

La nostra ipotesi è che un insegnamento esplicito della logica, in un ambiente di gioco che integra vari artefatti e vari registri interpretativi, aiuti non solo a migliorare la propria abilità matematica ma che favorisca lo sviluppo di abilità cognitive a priori indipendenti dalla logica matematica.

Per verificare le nostre ipotesi abbiamo analizzato l’esperienza didattica condotta in due scuole (guic e chat). Nella scuola primaria guicciardini mostriamo evidenza di come in effetti l’esperienza didattica aumenti le abilità cognitive sugli studenti trattati, comparando le performance a un test di matrii di Raven, ripetuto prima e dopo l’esperienza didattica in una classe trattata e in una di controllo. Per testare la matematical literacy abbiamo invece usato domande prese da test invalsi di anni precedenti (stavano in seconda non avevano mai fatto test invalsi). In entrambe le scuole abbiamo anche condotto un’analisi qualitativa sull’affect e engagment studenti nell’attività, per validare anche l’argomentazione che l’insegnamento della logica migliori la percezione della matematica.

**Methodological background**

**Come bul sviluppa le abilità cognitive e la matematical literacy: alternanza vero e falso, approccio all’errore, metalinguaggio, introduzione dei simboli. Per ognuno, argomentazioni di altri, applicazione dell’argomentazione nel caso di bul.**

The main methodology for the activities here described stems from a game in which students play the role of knights and knaves. The island of knights and knaves is a well known tale by Raymond Smullyan, describing an island whose inhabitants are either knights, who always tell the truth, or knaves, who always lie.

This island is mostly known from logical puzzles in which one is tasked with identifying the inhabitants of the island on the basis of their statements, with such puzzles making an appearance as early as primary education (Carotenuto et al., 2017).

Nonetheless, a story built around characters who speak the truth and characters who lie is likely to have a wider didactical value, not just because <<putting these matters in human terms has an enormous psychological appeal>> (Smullyan, 1987), but also because—owing to the use and acceptance of falsehoods (those uttered by the knaves)—it offers a more playful approach to errors, which become part of the learning process rather than being immediately corrected. Note that errors in mathematics can correspond to two very different situations: a syntax error, where the symbols are used incorrectly (e.g. 3 + + + = =), and a semantic error, where the symbols are used correctly but the meaning is incorrect (e.g. 3 + 3 = 5). A statement written with incorrect syntax does not have a truth value (it is neither true nor false), whereas a syntactically correct statement can be true or false.

Knights and knaves allow us to discuss semantic errors with ease, given that the errors made by knaves in their statements are always semantic, not structural. Generally, we believe that false statements can be used to improve understanding of various symbols and concepts: knowing that 3 < 2 is false helps to clarify the meaning of the symbol <; moving forwards, a livelli scolastici più avanzati, false statements can be useful in the context of formulating conjectures and finding counterexamples, particularly in the Game Semantics (in which proofs correspond to winning strategies in a dialogue between two contestants: Proponent and Opponent (Barrier, 2008) (Arzarello & Soldano, 2019).

However, false statements are often firmly avoided in education: as maintained by (Zan & Di Martino, 2017) in an approach commonly found in mathematical teaching that is focused on training—and thus focused on reproducible processes—error is synonymous with failure and, thus, to be avoided. However, avoidance of error limits the possibility of dialogic learning, based on the comparison between truth and falsehood. Recent research has involved incorrectly completed exercises being introduced for the purpose of student-conducted error analysis (Rushton, 2018). Error analysis consists of being presented with a problem statement alongside the steps taken to reach a solution, where one or more of the steps are incorrect. Students analyse and explain the errors and then complete the exercise correctly, providing reasoning for their own solution. Studies show an increased mathematical understanding when these practices are used with a combination of correctly and erroneously worked exercises (citazione? E approfondimento). A nostro avviso, risultati simili possono essere raggiunti usando le affermazioni corrette ed errate di cavalieri e furfanti. Come e perché?

Non solo per i motivi sopra citati (alternanza frasi corrette e frasi false) + raagionamento sul linguaggio + ragionamento per simboli.

In the classroom, students are rarely allowed to roam freely through the world of mathematics. Such roaming entails trying and failing (the word ‘error’ is derived from the Latin word *errare*, to wander or stray), and then modifying your approach on the basis of the information you acquire. When exploring primary school teachers’ opinions on logic, (Bibby, 2002) found the majority believe <<the objectivity of logic contrasts with the apparent subjectivity of the creative process>>, viewing logic as an obstacle to mathematical discovery. This belief seems to be based on a limited view of logic, in which logic is reduced to a syntactic formalism without semantic value and, above all, is considered a technique solely related to deduction, <<assumed as an unproblematic foundation for the justification of knowledge>> (Ernest, 1994). We believe that logic has a broader scope and can aid in the art of discovery.

**Trial run**

**Illustrare lo svolgimento dell’attività con riferimento ai 4 canali citati prima (anche registrazioni ed pareri personali ed esempi ed immagini e analisi di compiti).**

The activities presented here have been used in two second grade classes (ages 7-8) of the Guicciardini primary school in Rome, to introduce students to propositional logic [Bul], and in a CM1 class (ages 9-10) of the Lycée Chateaubriand in Rome, to illustrate propositional logic.

**Description of the activities**

In the following descriptions, we refer to both the researchers and the class teachers who participated as teachers.

*Phase 1 Guicciardini: theatral activity*

The aim of the first activity is to introduce students to the knight and knave characters. Masks for the two character types were prepared in advance, to be given to each student when needed.

The knight character was introduced first, described as someone who always tells the truth: after being given an example by the teacher, the students were then encouraged to make true statements — on any topic — while wearing a knight mask. Interestingly, the students immediately grasped the concept of a true statement: they began by providing statements about their immediate surroundings, and then moved on to general truths. Some statements were not verifiable (e.g., <<my sister is called Maria>>), but they all had a truth value. No students suggested phrases that did not correspond to a statement, such as “the umbrella”, nor phrases without a truth value, such as “it will rain tomorrow”. The students were then encouraged to come up with mathematical statements, once the teacher had provided an initial example of a simple calculation such as <<$2$ plus $2$ is equal to $4$>>.

The knave character was then introduced as someone who always lies, and — as with the knight — the students were encouraged to make false statements, first generally and then in a mathematical context.

Although the teacher’s initial examples were of incorrect calculations, such as <<2 plus 3 is equal to 2>>, some students opted for diverse examples of mathematical falsehoods, such as <<100 has two figures>>.

The next part of the activity was more similar to Smullyan’s classic puzzles, where students had to work out whether the character speaking was a knight or a knave. This activity required two teachers, one who wore a knight or knave mask with their back turned to the class and provided riddles for the students, and another who helped the students to solve the riddles and identify which character was speaking. To start, the riddles were very simple, as they did not follow the classic formulation seen in Smullyan’s puzzles (which self-refer to the same group of characters speaking) but were simple statements such as <<tigers can fly>>. The students were then asked in turn to assume the role of the knight or knave and provide riddles for their classmates. The teacher then introduced the emblematic statement <<I am a knight>>, always with their back turned to the class. After initial attempts to reach a decisive solution — during which both characters were suggested — the class realised that it was not possible to know whether the person speaking was a knight or a knave on the basis of that statement alone.

Similarly, the class was encouraged to consider the phrase <<I am a knave>>, and were pleased to discover that neither character would have been able to say this phrase. To finish off the first part of phase 1, some of Smullyan’s simpler, classic riddles were proposed to the class, which included more than one masked character with their backs to the class (one teacher and one or more students). The teacher told each of the masked students what to say. It is important to note that creating physical representations of the characters making these statements — with their backs turned and faces hidden, but nonetheless there in person — is likely to have made it easier for the students to solve the riddles.

To finish off the activity, the class were introduced to Boolean circuits, using the knave as a representation of ‘false’ and the knight as a representation of ‘true’. This choice works on a logical level, given that for every statement $A$ made by a knave, we have $A \leftrightarrow FALSE$ , and for every statement $B$ made by a knight, we have $B \leftrightarrow TRUE$.

The aim in each circuit is to reach the final circle — shown in red in the figure — while wearing a knight mask. At the start of the circuit (in the blue circle), the player chooses which mask to wear. The first circuit proposed is trivial, whereby the player simply has to follow the rope to the finish circle—with no unexpected events along the way.

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The second circuit introduced Mr No, a character (played by a student) who forces any player who encounters him to change their mask. The winning strategy, as shown in the figure, is to start the circuit wearing the knave mask.

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The next circuit then included two Mr No’s, one after the other; here, the winning strategy is to start the circuit wearing the knight mask. More Mr No’s were then introduced sequentially into the circuits, leading towards a discussion on the parity of the number of negations: if there is an even number of Mr No’s—including none at all—the winning strategy is to start with the knight mask; if there is an odd number of Mr No’s, the winning strategy is to start with the knave mask.

*Phase 1 Chateaubriand: theatrical activity*

The activity with the fourth-grade class followed the same steps as with the second-grade class, but at a quicker pace. Notably, a student made a statement that triggered the discussion of a key point, declaring <<we are all girls>> while wearing a knave mask; this statement opened the floor to a discussion on the terms ‘all’, ‘at most’, ‘at least’, and ‘none’. When initially questioned, the students fell foul to the classic mistake of thinking the negation of ‘all’ to be ‘none’, and vice versa. To confront this, the teacher laid a pink hula hoop on the floor and said <<all of the circles are pink>>; the class correctly identified the teacher as a knight. After this, the teacher put another pink hoop on the floor and repeated the phrase. Again, the class correctly identified them as a knight. The teacher added yet another pink hoop to the collection, and the discussion was repeated. Finally, the teacher laid down a green hoop: at this point, there were three pink hoops and one green hoop on the floor. The teacher then said <<all of the circles are pink>>, at which point the class correctly identified them as being a knave this time. In this way, the class was able to explore the key concept by which the negation of ‘all’ is ‘there exists one that is not’. Knights and knaves are a useful tool for the analysis of such words: negation of quantifiers can be hard to grasp, but identifying with a character who lies or speaks the truth can aid in understanding. Through these conversations, the students began to discuss elements of set theory.

This first phases lasted around 1 hour and 30 minutes. To consolidate the concepts learned in this phase, and to approach them from another perspective, the classes played the Bul Game before moving onto phase 2. Bul Game is an online game in which players encounter either a knight or a knave in each turn, and need to choose between two options on the basis of a statement given by the knight or knave in order to progress through the game. The types of riddles encountered are classified into sections that reflect the stages of the activities. Before phase 2, all the classes played the first section of BUL GAME, TRUE&FALSE.

*Phase 2 Guicciardini: predicates*

The students were told that knights and knaves sometimes use a strange way of writing. First of all, they were asked to pick out the key elements of a phrase such as “a tiger is an animal”, identifying ‘tiger’ and ‘animal’ as essential words to understand its meaning. Anticipating the next step, a student noted that <<a ‘not’ would be important too if it were there>>. The students were then given parentheses to colour in, to familiarise them with the symbol. The students were then told that knights and knaves could use the two words ‘tiger’ and ‘animal’ and parentheses to write the phrase “a tiger is an animal”. Some students suggested TIGER(ANIMAL) as a potential solution, and others (TIGER ANIMAL) (which is somewhat reminiscent of Barandrecht’s lambda calculus!); a few other students suggested ANIMAL(TIGER). Each of these three notations can be used without leading to contradictions. The students were finally told that knights and knaves use the notation ANIMAL(TIGER). This is the standard notation used in logic and general mathematics, where the object of the predicate, or function, sits within parentheses after the symbol for the function.

We feel that this early introduction of formal notation can be beneficial: first, as mentioned in the introduction, it allows students to become accustomed to using a symbolic and context-dependent language (this language is used exclusively on Smullyan’s island of the knights and knaves). This situation highlights the fact that changing language does not necessarily involve changing the vocabulary or alphabet; rather, the same words and the same symbols can be used in a different way without changing the meaning of the phrase — namely, by translating. The key point is to create a broader view of language, which is not defined exclusively by its alphabet and vocabulary, but also by the rules that govern the construction of phrases. Furthermore, notation such as ANIMAL(TIGER) encourages an intensive rather than extensive description of the property of being an animal — this point will be further discussed in what follows. Finally, the formal structure of predicates makes it easier to write phrases with the negation symbol, as we will see in the following phases.

Great importance is placed on the translation from symbolic form to natural language: TREE(OAK) should not be read <<tree oak>> but <<the oak is a tree>>. The students were given statements to translate in both directions, with examples of true statements — i.e., those made by a knight, such as ANIMAL(TIGER) — and false statements — i.e., those made by a knave, such as ANIMAL(TABLE). A further exercise was to complete predicates correctly on the basis of who was speaking—for example, EVEN(…) or $3 < $… .

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*Phase 3 Guicciardini: negation*

The phrase <<a tiger is not an animal>> was written on the board and the students were asked, as in phase 2, to identify the key words. At this point, it was noted that, in addition to ‘tiger’ and ‘animal’, the word ‘not’ was also fundamental. A few exercises were done on the board whereby students needed to work out whether a given phrase had been said by a knight or a knave; for example, the first phrase was said by a knave, whereas the phrase <<$3$ is not even>> was said by a knight. The class was then given the negation symbol $\neg$ to colour in, to familiarise them with the symbol. Many recognised the symbol from the first phase, when it was used with the Mr No character. Following this, the students carried out translation exercises — first, orally at the board, and then written — and were given comics to fill in, depending on whether the person speaking in the comic was a knight or a knave. The phrase <<red is not a colour>> would be translated as $\neg$COLOUR(RED). Similarly, the phrase $\neg$ODD($4$) is translated as <<4 is not an odd number>>.

It should be emphasised here that two different approaches were taken for the negation symbol. In phase 1, the symbol was introduced as a rule—the symbol acted on the truth value of a statement by changing it, that is by changing the mask worn—whereas in phase 3, the negation symbol was introduced as a logical connective with semantic value. These two interpretations are clearly very closely connected. If either a knight or a knave writes the phrase PREDICATE(OBJECT), then the introduction of the negation symbol will force a character swap, because the phrase $\neg$PREDICATE(OBJECT) could only be written by the other character.

Until this point, the statements provided that contained the negation symbol had been limited to the form $\neg$PREDICATE(OBJECT). The statement $\neg (3 < 2)$ was then written on the board and a student was asked to translate it. Surprisingly, the student translated it as <<$3$ is not less than $2$>>, thus putting the negation before the predicate.

It is worth noting that frequently, in natural language, the position of a negation is not well defined a priori; in some cases, moving its position does not affect the meaning of the phrase (e.g. <<all people do not have blonde hair>> is equivalent to saying <<all people have non-blonde hair>>), whereas in other cases, moving the negation can distort the meaning (e.g. <<not all people have blonde hair>> is completely different from <<all people do not have blonde hair>>). Further examples relate to double negations, which work as affirmations in some languages and negations in others. We believe that introducing the negation symbol and its rules aids in the understanding and clarification of these various situations.

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*Phase 2 Chateaubriand: predicates and negation*

For the fourth-grade class, phases 2 and 3 of the activities were merged. Furthermore, binary predicates were introduced — i.e., predicates involving two objects. The first binary predicate to be introduced was MOTHER($x$, $y$), with the teacher going through several examples with the students; the chosen convention was that $x$ is the mother of $y$. Examples of this predicate were given where a knight was speaking, as well as where a knave was speaking. The predicate FRIENDS($x$, $y$) was then introduced, with further examples. Referring always to knights and knaves, it was noted that writing MOTHER($x$, $y$) is different to writing MOTHER($y$, $x$) (indeed, one case precludes the other), whereas writing FRIENDS($x$, $y$) is equivalent to writing FRIENDS($y$, $x$). This property of the latter was described to the class as symmetry, which is common in mathematics: the symbol $<$, for example, is not symmetrical whereas the equality symbol $=$ is. Finally, the similarities between symmetry and commutativity were highlighted to the class.

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*Phase 4 Guicciardini: variables*

Questa parte dell’esperienza riguarda l’analisi, nel senso che i bambini fanno analisi e avere il concetto di furfante aiuta ad affrontare l’analisi. Avere a mente il personaggio del furfante aiuta a non spaventarsi delle ipotesi sbagliate nelle sostituzioni. (forse mettere alla fine).

In the final phase, the variable $x$ was introduced as a mystery number. Questions such as <<I know a number $x$ such that $x + 3 = 8$. What number is it?>> or <<I know a number $x$ which, when added to itself, makes $10$. What number is it?>> were asked. It was not hard for the students to answer these first simple examples. Such equations, which are generally first encountered in middle school (ages 11–14 years), are usually solved using a synthetic method that relies on inverse operations. We naturally did not consider it appropriate to introduce such a method at primary school and the equations were instead solved in an analytical way, by trial and error: different numbers were substituted for $x$ on the board and the resulting equality was checked. The class was used to recognising false statements (and judging them as such) thanks to their familiarity with the knave character. We believe that solving through trial and error should also be encouraged in later year groups. More complex statements were then proposed to the class, such as EVEN($x$), and the class noted that, this time, there were many different possible solutions. Multiple requirements were therefore added together: <<I know a number $x$ such that EVEN($x$), $x < 10$, and $x$ is written with three letters. What number is it?>> In this case, there is still more than one solution, but the number is limited.

The main exercise in this activity involved laying out many numbers and formulas containing an $x$ on the ground. The aim was to complete the equations, inequalities, or predicates by placing an appropriate number over the $x$ — in other words, substituting a constant for a variable. It was a good example of wandering through mathematics.

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*Phase 3 Chateaubriand: sets and variables*

The terminology already discussed in phase 1, such as ‘all’, ‘at least one’, and ‘none’, was reviewed through examples, as was — with the help of knights and knaves — the concept that the negation of ‘all’ is ‘at least one is not’ and the negation of ‘none’ is ‘at least one is’. Sheets of paper depicting the elements of a set were then hung up, with an image of a knight and a knave placed on either side of the set. In turn, each student was asked to describe the set, first as a knight and then as a knave, with the only requirement being that they use one of the terms ‘all’, ‘at least one’, or ‘none’. The sets used were first of humans, then of geometrical shapes, and then of numbers.

The concept of a variable was then introduced, using similar examples to those used in the second-grade class. This concept was then incorporated into the previous activity. A formula was written on the board that contained x and the students were asked whether the elements $x$ that satisfied the formula (referring implicitly to integers) were all, some (i.e., at least one; the students did not seem to have a problem with it being exactly one), or none. For example, $x + 3 = 5$ is satisfied by one number, whereas $x = x$ is satisfied by all numbers; by contrast, $x > x$ and $x + 3 = 1$ is not satisfied by any integers. It was then noted that EVEN($x$) is satisfied by an infinite amount of numbers, as is ODD($x$). Indeed, it was pointed out that EVEN($x + x$) is satisfied by all integers! The class was thus asked to find an equivalent expression such that ODD(expression) was true for all integers. At first, the class had no idea how to approach this problem, but then began to work out what sort of expression would be required. They started working on a solution in small groups, supported by three teachers. The students suggested solutions such as ODD($x - x + 1$): they were told that while correct, these expressions always give the same result, regardless of the value of $x$, and that they should try to find a non-constant expression. After a while, several students, independently, concluded that a possible solution was ODD($x + x + 1$).

The rest of the activity followed the same stages as for the second-grade class, with numbers laid out on the ground, alongside formulas to be completed, with a few more complex examples introduced.

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**Empirical analysis**

In this section we explain how we assessed the causal impact of the BUL educational experience in the development of cognitive skills and mathematical literacy, in order to provide empirical evidence supporting our main argument.

**Methodology**

The empirical exercise has been structured as follows: we selected two different second year classes in Guicciardini primary school, in order to perform a randomized control trial in which BUL itself was used as treatment. While the treatment unit was the class in which the educational experience was undergone, as control unit we choose the other class of the same year sharing the same teacher, in order to avoid possible sources of endogeneity deriving from having a different instructor. After checking with proper balancing test that the class composition was as good as random with respect to relevant covariates (age, sex and nationality of origin), we measured mathematical literacy and cognitive skill in both classes, before and after the treatment. In order to measure mathematical literacy, we used a set of INVALSI questions from previous years. INVALSI are national tests specially designated and recognized by the Italian state to evaluate skills (understood as knowledge and ability to think about knowledge) in fundamental areas such as mathematics, Italian and English. A set of questions taken from the mathematics INVALSI tests therefore lends itself as a good measure of mathematical literacy.

In order to measure cognitive skills we used indeed a set of Raven’s progressive matrices, non verbal test broadly recognized as a measure general human intelligence (g factor) for individuals ranging from 5 years old to the elderly (Kaplan and Saccuzzo 2009). In particular, we used the variant of the Colored Progressive Matrices (RCPM), a version of the Raven test designed specifically for children aged 5 to 11 (Domino and Domino, 2006). We considered this kind of test a reliable proxy of the general cognitive skills, since a review of the evidence from psychological literature suggest that there is no factor of intelligence independent from the factor g (Jensen 2006).

It is also important to underline that RCPM are a non verbal test of cognitive skills, while the educational path implemented and logic itself has a strict connection with verbal reasoning, so the only effect captured by the exercise should come from an increase in general cognitive skills and not from an ad hoc training.

**Data**

The dataset used for this RCT consisted in two second year classes, whose students present in both tests made up the treatment and control group. After eliminating the outliers who scored the maximum in the first test - not being able to measure any improvements - the dataset was made up of 18 students per class.

The following table summarized the relevant charateristics of the two groups:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **Avg. RCPM score** | **N Males** | **N Foreigners** | **Tot students** |
| **Control** | 10,3 | 9 | 5 | 18 |
| **Treatment** | 10,2 | 11 | 5 | 18 |

We included in the relevant characteristics all the observables that can affect the improvementin the tests. Children were all of the same age, so we decided to perform balancing tests for sex, nationality of origin and initial RCPM score. Being a measure of fluid intelligence, and given the relationship between learning and intelligence stated in the above mentioned Jensen (2006), we allow for the possibility of initial cognitive skill affecting the rate of learning of both mathematical literacy and cognitive skill, therefore we included the initial RCPM score in the relevant characteristics. Also according to Vaci et al. (2018), the more intelligent people benefit more from practice, thus for example a child with higher initial cognitive skills can improve more his mathematical literacy just from the standard math classes.

Then we performed balancing t-test on these main characteristics in order to validate the assumption of class composition being as good as random.

We used a Student’s t-test for mean differences since variance of each covariate between the group was similar, and since it can be used also for very small samples (de Winter 2013).

The results are showed below:

|  |  |  |  |
| --- | --- | --- | --- |
|  | **RCPM** | **Foreigner** | **Sex** |
| **t-stat** | **-**0,23 | 0 | 1,07 |
| **p-value** | 0,42 | 0 | 0,62 |

The results show no significant difference between the two groups regarding the relevant observables, validating the assumption of class composition being as good as random. An important issue that we could not solve, and so we will consider when discussing the results, is the influence of the memory effect. Since the return RCPM test was made by the same questions that composed the first one, it is possible to observe a memory effect bias, even if the students were never given with the solutions of the tests. **Results**

The above mentioned balancing tests allowed us to directly consider the coefficient of the regression of the treatment dummy on outcome variable – that measured the score difference between the first and the return test for both mathematical literacy and cognitive skill.

The result for the cognitive skill are shown below:

|  |  |
| --- | --- |
| **Coefficient** | 0,55\*\* |
| **Standard Error** | 0,29 |
| **p-value** | 0,06 |

While here the results for mathematical literacy are reported:

|  |  |
| --- | --- |
| **Coefficient** | 2,27**\*\*** |
| **Standard Error** | 0,88 |
| **p-value** | 0,09 |

As we can see from the results, the effect of the treatment is significant on both cognitive skill and mathematical literacy. Concerning cognitive skills, the coefficient is smaller but it has higher significance; for mathematical literacy, indeed, it is larger but with lower significance level. This can be due to the presence of memory effects, as discussed above. Moreover, as argued in the conceptual framework, the study of logic is more likely to have a larger impact on mathematical literacy, ( discuss together)

**Discussion**

Please use 10-point font size. Please margin the text to the justified. Manuscripts should be 1.5 times spaced. A paragraph should have at least 3 sentences. Footnotes and endnotes are not accepted. All relevant information should be included in main text. Do not indent paragraphs; leave a space of one line between consecutive paragraphs. Do not underline words for emphasis. Use italics instead. Both numbered lists and bulleted lists can be used if necessary. Before submitting your manuscript, please ensure that every in-text citation has a corresponding reference in the reference list. Conversely, ensure that every entry in the reference list has a corresponding in-text citation.

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**Conclusion**

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**Recommendations**

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**Acknowledgements or Notes**

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