The Endowment Effect via Social Learning: Model

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1 Instroduction

I first set out an illustrative example with discrete value levels and concrete probabilities (Section 2). And then I present a model that expands that illustrative example (Section 3). The idea of the model is also displayed graphically in Figures 1-3.

2 An Illustrative Example

Suppose there are two traders, a buyer and a seller, each of whom have to decide what price they are willing to trade at, and the indivisible good to be traded can either be of high value or low value, and let us set those to 0 and 1 respectively. Furthermore, it might be 0 to one of the traders and 1 to the other. Thus, there are four possible states of the world, and let us set those probabilities such that if the good is of high value to the seller it is more likely it is also of high value to the buyer (see Table 1).

	$u_b = 0$	$u_b = 1$
$u_s = 0$	$\frac{1}{3}$	$\frac{1}{6}$
$u_s = 1$	$\frac{1}{6}$	$\frac{1}{3}$

Table 1: Prior probability distribution of the four states of the world. If the seller values the good highly then it is more likely that the buyer also values the good highly ($\mathbb{E}(u_s) = \frac{1}{2}$).

		$u_b = 0$	$u_b = 1$
$\rightarrow$	$u_s = 0$	$\frac{4}{9}$	$\frac{2}{9}$
	$u_s = 1$	$\frac{1}{9}$	$\frac{2}{9}$

Table 2: Posterior probability distribution of the four states of the world, given the seller has received a low signal ($\mathbb{E}(u_s) = \frac{1}{3}$).

Neither buyer nor seller know their own values, but each receives an independent, noisy, and private signal either signalling that the good is worth 0 (a low signal) or 1 (a high signal). Each signal has a $\frac{2}{3}$ chance of being correct, such that if the seller receives a low signal, their personal expected value of the good is $\frac{1}{3}$. A naive seller might therefore be willing to sell at any price above that expected value of $\frac{1}{3}$. A rational seller, however, would only be willing to trade at a price that is above their expected value *given* that the buyer is willing to buy at that price.

If the buyer is willing to consider a price above $\frac{1}{3}$, then they must have received a high signal.[1] Armed with this information, the seller can Bayesian update (see Table 3) resulting in a new expected value (to the seller) of $\frac{5}{13}$. Likewise, the buyer knows that if the seller is willing to consider a price below $\frac{2}{3}$, then they must have received a low signal, and the buyer can Bayesian update resulting in a new expected value (to the buyer) of $\frac{8}{13}$.

[1] A buyer with a low signal would not consider the price of $\frac{1}{3}$ because they would correctly conclude that if the seller was willing to sell at $\frac{1}{3}$ then they must have received a low signal too and if both traders received a low signal their expected value would only be $\frac{4}{14}$.

		$\downarrow$
	$u_b = 0$	$u_b = 1$
$\rightarrow$ $u_s = 0$	$\frac{4}{13}$	$\frac{4}{13}$
$u_s = 1$	$\frac{1}{13}$	$\frac{4}{13}$

Table 3: Posterior probability distribution of the four states of the world, given the seller has received a low signal and the buyer has received a high signal ($\mathbb{E}(u_s) = \frac{5}{13}$). The seller updates their valuation from $\frac{1}{3}$ to $\frac{5}{13}$ when they learn the buyer is willing to buy and therefore must have recieved a high signal.

Thus, the rational seller's WTA is greater than their prior expected value of the good, and the rational buyer's WTP is lower than their prior expected value of the good. The key properties that cause this WTP-WTA gap are (1) that the trader is uncertain about the value of the good, and (2) that their value of the good is affiliated to their trading partners. Furthermore, the greater of either of those two properties (uncertainty and affiliation), the greater the gap.

The following section expands this example into a more general model.

3 Model

One seller owns one unit of an indivisible good, which they value at v_s. The buyer values the good at v_b. v_s is the sellers type, and v_b is the buyers type. Neither player knows their own type or the type of the other player. The players types are jointly distributed on $\mathbb{R}^2$ according to the joint distribution, F_v. It might be helpful to think of v_i as made up of a common component, $\bar{v}$ and idiosyncratic component, u_i, that is $v_i = \bar{v} + u_i$. [2] x_s is the signal observed by s and x_b is the signal observed by b. Each signal is unbiased, $x_i = v_i + \epsilon_i$ with $\mathbb{E}(\epsilon_i) = 0$, and distributed according to F_ϵ. To keep it simple, ϵ_s and ϵ_b are independent draws from the same distribution, but they could also be from different distributions.

Variable	Description
v_s	Unobserved value to seller, s
v_b	Unobserved value to buyer, b
F_v	Joint Distribution of v_s & v_b
x_s	Observed signal of v_s $= v_s + \epsilon_s$
x_b	Observed signal of v_b $= v_b + \epsilon_b$
F_ϵ	Distribution of ϵ_i
p	price
π_s	Seller Payoff $= 0$ if no trade $= p - v_s$ if trade
π_b	Buyer Payoff $= 0$ if trade $= v_b - p$ if trade

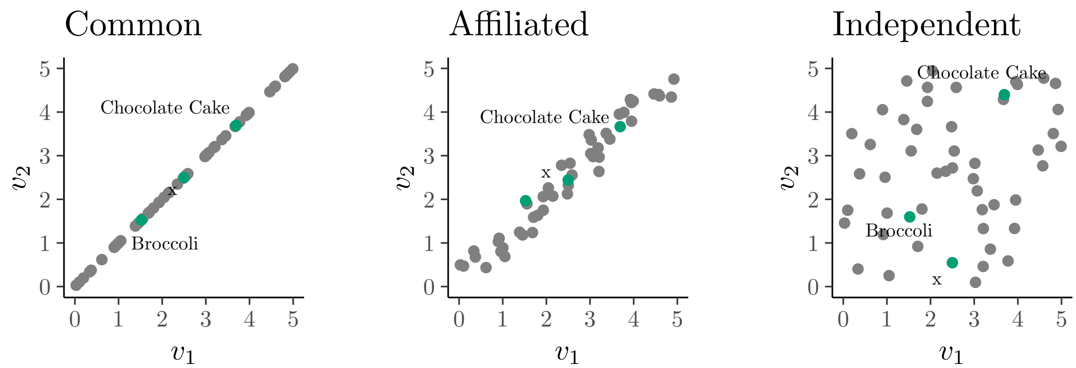
Nature moves first and chooses v_s, v_b, and the price, p. p is an independent random variable. Then nature chooses ϵ_s and ϵ_b, which determines x_s and x_b respectively. b observes p and x_b, and decides if they are willing to buy the good at p. Likewise, s observes p and x_s, and decides if they are willing to sell at p. If either b or s are unwilling to trade then each receives a payoff of 0. If both are willing to trade then b receives $v_b - p$, and s receives $p - v_s$.

3.1 Equilibrium

For convenience, let us restrict ourselves to cases where p falls between x_b and x_s. There are two assumptions, either one of which is sufficient for there to be no gap, that is WTP $= x_b$ and WTA $= x_s$.

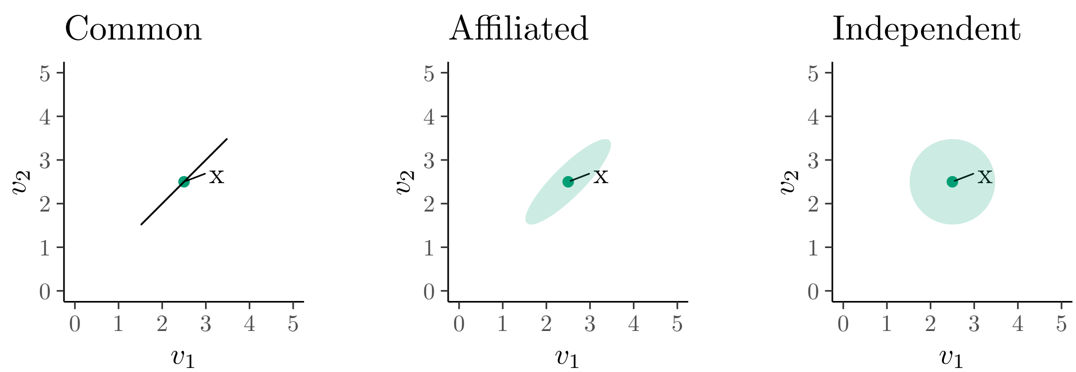
[2]However, $\bar{v}$ is *not* common knowledge. If it were common knowledge there would be no gap.

Figure 1: Affiliation



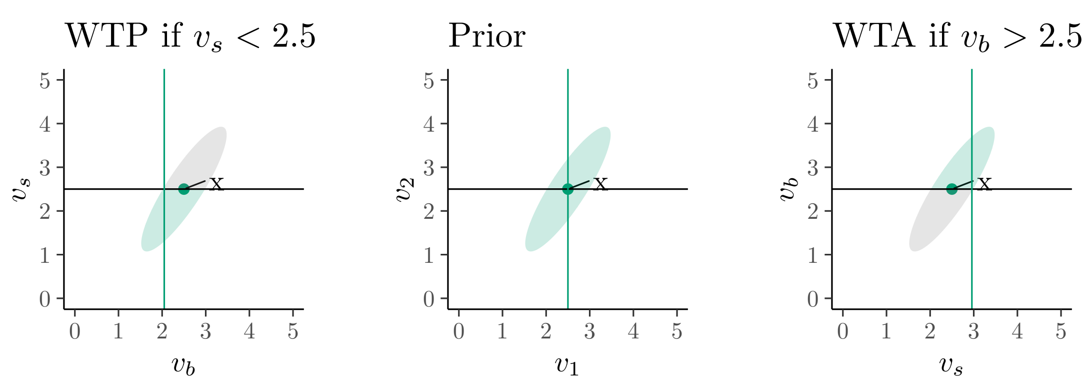
Notes: The left panel shows the values of 50 goods for two people who share exactly the same utility function (common value). The right panel shows two people whose values are independent (private value). The middle panel shows two people whose utility is correlated (affiliated value). It is sometimes written $v_i = \bar{v} + u_i$ where $\bar{v}$ is the common element and u_i is the idiosyncratic or private element.

Figure 2: Uncertainty: Joint Distribution of v_s and v_b for One Good.



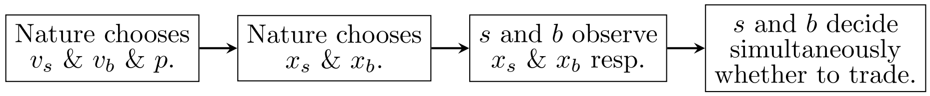
Notes: The ellipse shows the 95% probability limit for the joint distribution of v_s and v_b. For common value utility functions, if you know one you know the other. For independent utility functions, if you know one, it tells you nothing about the other. For affiliated utility functions, if you know know one, it tells you something about the other.

Figure 3: Expected Value shifts if the buyer/seller gets info. about their trading partner's valuation.



Notes: In the left panel the x-axis person is the buyer. If she learns that the seller values the good at lower than 2.5. This means that her posterior distribution shifts downwards, along with her expected value of the good, that is, her WTP. The middle panel shows the prior distribution. The right hand panel shows the symmetric case where the the x-axis person is the seller. Of course, in the model the buyer may not learn that seller's $v_s < 2.5$, but she knows that the seller will only sell if $x_s < p$, and she knows p which tells her something about x_s, and therefore, v_s.

Figure 4: Order of Play



These are also the only ways to achieve ex-ante efficiency.[3] Firstly, if F_v is independent, then there is no gap. Secondly, if ϵ has zero variance (and therefore $\epsilon = 0$), that is i knows v_i, then there is also no gap. If independence and certainty are both relaxed, then there will be a gap and the outcome is not ex-ante efficient. In other words, if $WTP(x)$ is the the buyer's maximum amount they are willing to pay as a function of their signal, and WTA is the seller's minimum amount they are willing to accept as a function of their signal, then $WTP(\alpha) < WTA(\alpha)\ \forall \alpha$.

The gap between $WTP(\alpha)$ and $WTA(\alpha)$ depends on F_v and F_ϵ. Firstly, the gap is bigger if the buyer and seller are more affiliated, that is there is a stronger correlation between v_s and v_b. Secondly, the gap is bigger if there is higher variance in ϵ, that is the more uncertainty that traders have in v_i.

3.2 Predictions

Greater uncertainty and greater similarity with respect to the good increase the gap size.

	Predictions
Information	High information goods (often traded and familiar to the traders) have smaller gaps. Low information goods (not traded and unfamiliar) have bigger gaps.
Experience	Experienced traders should have smaller gaps. First time traders have bigger gaps. Gaps decrease as traders increase experience.
Comparable goods	In the exchange paradigm[4] goods that are similar / easier to compare have smaller gaps.
Market Thickness	Bilateral trade has bigger gaps, thick markets leads to smaller gaps.
Trading Partner	Similar trading partners (peers) leads to bigger gaps, dissimilar trading partners (firms) lead to smaller gaps.

Table 4

[3] I mean here by 'Ex-ante efficiency' the property such that for all possible values of the signals x_b and x_s, if $x_s < x_b$ then trade will happen, and else trade will not happen