Social efficiency vs. social equity

Peter and Samuel

June 2021

Consider an agent i evaluating a set of payoffs among people involving herself and others: $\{x_1, \ldots, x_N\}$, where $i \in \{1, \ldots, N\}$. Suppose the agent considers her own payoff x_i; the social mean, $\bar{x} = \frac{1}{N}\sum_j x_j$; and the social dispersion, $\sigma = \sqrt{\frac{1}{N}\sum(x_j - \bar{x})^2}$. Suppose her preferences are parameterized by,

$$u_i(x_i, \bar{x}, \sigma | \boldsymbol{\alpha}, \boldsymbol{\omega}) = (1-\alpha_i)x_i + \alpha_i N\bar{x} - \omega_{a,1i}\mathbb{1}\{x_i \geq \bar{x}_{-i}\}\sigma - \omega_{a,2i}\mathbb{1}\{x_i \geq \bar{x}_{-i}\}\sigma^2$$
$$- \omega_{b,1i}\mathbb{1}\{x_i < \bar{x}_{-i}\}\sigma - \omega_{b,2i}\mathbb{1}\{x_i < \bar{x}_{-i}\}\sigma^2,$$

where $\boldsymbol{\alpha}_i$ represents i's preference weight on "efficiency" and $\boldsymbol{\omega}_i$ on "equity."[1] This specification nests Bellemare et al (2008) and Fehr and Schmidt (1999) as special cases.

Suppose now we ask agent i to choose allocations between herself and j in one game; and among herself, j and k in another two games. For example, let $\alpha_i = 0.75$, $\omega_i = \omega_{p,2i} = \omega_{r,2i} = 0.5$, and $\omega_{p,1i} = \omega_{r,1i} = 0$. Consider the following three sets of allocations and associated decision utilities:

Table 1: Two-person game: allocations and utilities

x_i	x_j	$\Rightarrow$	u_i	u_j
10	0		-2.5	-5
9	1		1.75	-0.25
8	2		5	3.5
7	3		7.25	6.25
6	4		8.5	8
5	5		8.75	8.75
4	6		8	8.5
3	7		6.25	7.25
2	8		3.5	5
1	9		-0.25	1.75
0	10		-5	-2.5

In the two-person game, i solves: $\max_{x_i}(1-\alpha_i)x_i + 10\alpha_i - \frac{\omega_i}{4}(2x_i - 10)^2$; with solution x^*: $1 - \alpha_i = \omega_i(2x_i^* - 10)$.

In the three-person game with a positive externality, i solves: $\max_{x_i}(1-\alpha_i)x_i + \alpha_i(20-x_i) - \frac{2\omega_i}{9}(2x_i - 10)^2$; with solution $x^\dagger$: $1 - 2\alpha_i = \frac{8\omega_i}{9}(2x_i^\dagger - 10)$.

In the three-person game with a negative externality, i solves: $\max_{x_i}(1-\alpha_i)x_i + \alpha_i(10+x_i) - \frac{2\omega_i}{9}(2x_i - 10)^2$; with solution $x^\ddagger$: $1 = \frac{8\omega_i}{9}(2x_i^\ddagger - 10)$.

The discrete solutions can be read directly from the three tables above (the value of x_i that generates the highest u_i). As can be seen, identification of parameters is straightforward.

[1] These concepts are simplified, here, for tractability, boiled down from a general social preference i given a community of N individuals, $u_i(x_1, \ldots, x_N)$.

Table 2: Three-person game with a positive externality

x_i	x_j	x_k	$\Rightarrow$	u_i	u_j
0	10	10		3.89	10.17
1	9	9		7.39	11.81
2	8	8		10.00	12.86
3	7	7		11.72	13.33
4	6	6		12.56	13.21
5	5	5		12.50	12.50
6	4	4		11.56	11.21
7	3	3		9.72	9.33
8	2	2		7.00	6.86
9	1	1		3.39	3.81
10	0	0		-1.11	0.17

Table 3: Three-person game with a negative externality

x_i	x_j	x_k	$\Rightarrow$	u_i	u_j
0	10	0		-3.61	-1.11
1	9	1		1.39	3.39
2	8	2		5.50	7.00
3	7	3		8.72	9.72
4	6	4		11.06	11.56
5	5	5		12.50	12.50
6	4	6		13.06	12.56
7	3	7		12.72	11.72
8	2	8		11.50	10.00
9	1	9		9.39	7.39
10	0	10		6.39	3.89

References

[1] Björn Bartling, Roberto A. Weber, and Lan Yao. Do Markets Erode Social Responsibility? *. *The Quarterly Journal of Economics*, 130(1):219–266, February 2015.

[2] Charles Bellemare, Sabine Krger, and Arthur van Soest. Measuring Inequity Aversion in a Heterogeneous Population Using Experimental Decisions and Subjective Probabilities. *Econometrica*, 76(4):815–839, July 2008.

[3] Gary Charness and Matthew Rabin. Understanding Social Preferences with Simple Tests*. *The Quarterly Journal of Economics*, 117(3):817–869, August 2002.

[4] Stefano DellaVigna. Chapter 7 - Structural Behavioral Economics. In B. Douglas Bernheim, Stefano DellaVigna, and David Laibson, editors, *Handbook of Behavioral Economics: Applications and Foundations 1*, volume 1 of *Handbook of Behavioral Economics - Foundations and Applications 1*, pages 613–723. North-Holland, January 2018.

[5] Stefano DellaVigna, John A. List, Ulrike Malmendier, and Gautam Rao. Estimating Social Preferences and Gift Exchange at Work. Technical Report 22043, National Bureau of Economic Research, Inc, February 2016. Publication Title: NBER Working Papers.

[6] Ernst Fehr and Klaus M. Schmidt. A Theory of Fairness, Competition, and Cooperation. *The Quarterly Journal of Economics*, 114(3):817–868, August 1999.

[7] Daniele Girardi, Sai Madhurika Mamunuru, Simon Halliday, and Samuel Bowles. Does Economics Make You Selfish? *UMass Amherst Economics Department Working Paper Series*, January 2021.

[8] Mark Ottoni-Wilhelm, Lise Vesterlund, and Huan Xie. Why Do People Give? Testing Pure and Impure Altruism. *American Economic Review*, 107(11):3617–3633, November 2017.