# Power Analysis for Narrative WTP (BDM)
# 3-Signal Bundle, Technology 0.45/0.15/0.40

## Design Summary

We study a $2 \times 2$ *between-subjects* design (Framing $\times$ Signal Technology).In this power analysis, we posit that only the Narrative $\times$ Nondiagnostic (Treatment from now on) condition increases mean WTP; the other three treatment conditions have no effect on WTP (i.e., their mean equals control).

We compare **Baseline** vs. **Treatment** on the *willingness to pay* (WTP, in points) for a bundle of **three** signals, elicited via a BDM mechanism. Ex-ante signal probabilities are the same across conditions and they are common knowledge.

## Setup, Assumptions and Theoretical Treatment Difference

### Setup

- **Signal technology.** For each bundle, we draw three signals $s \in \{\text{pos}, \text{neg}, \text{nd}\}$ independently conditional on the true state. Per draw, the probabilities are (0.45, 0.15, 0.40). Here, pos denotes a signal aligned with the true state, neg a signal that contradicts it, and nd a nondiagnostic (or missing) signal.

- **Scoring rule:** Quadratic scoring rule (Brier).

### Assumptions

- **Beliefs**: Let $\theta$ be the true state of the world. Prior $p = \Pr(\theta = 1) = 0.5$; Bayes rule to compute the EVSI[1].

- **Baseline (C_No_Info, C_Yes_Info, N_No_Info)**: Subjects value the bundle by the theoretichal EVSI under 0.45/0.15/0.40:

$$\text{WTP}_{\text{control}} = 8.916 \text{ points.}$$

### Treatment Difference

- **Treatment (narrative-boosted nondiagnostic).** We model the narrative as making nondiagnostic signals partially informative *in perception*. Operationally, we halve the nondiagnostic probability and redistribute the freed mass to pos and neg in the same 3:1 ratio as their baseline shares, giving (pos, neg, nd) = (0.60, 0.20, 0.20). Signals remain conditionally independent (given the state) with three draws per bundle. This leads to:

$$\text{WTP}_{\text{treat}} = 11.069 \text{ points.}$$

[1] Expected Value of Sample Information, more details in Appendix A.

Hence the **theoretical treatment difference** between these technology is:

$$\Delta \equiv \text{WTP}_{\text{treat}} - \text{WTP}_{\text{control}} = 11.069 - 8.916 = 2.152 \text{ points},$$

corresponding to an effect size of approximately

$$d = \frac{\Delta}{\sigma} = \frac{2.152}{5} \approx 0.43,$$

which can be interpreted as a **moderate effect** according to conventional benchmarks.

## Power Calculations

We test the difference in mean WTP between treatment and baseline using a two-sample $t$-test with equal variances ($\sigma = 5$), two-sided, at $\alpha = 0.05$ with target power $1 - \beta = 0.80$.

We compare mean WTP between treatment and baseline at the participant level (one observation per participant, defined as the average WTP over 15 rounds), treating participants as independent. For power calculations, we account for within-participant correlation across rounds via the design effect

$$DE = 1 + (m-1)\rho,$$

with $m = 15$ and $\rho \in \{0.3, 0.4, 0.5\}$, which maps per-round variability into the variance of the participant-level mean used in the test.

The unit of analysis is the participant-level mean WTP (averaged across the 15 rounds). Power calculations assume independent observations across participants and common SD $\sigma = 5$ per arm (conservative). As a robustness check, we will run 1000 simulations using a panel regression with the expected effect size and the planned number of observations per treatment.

For equal variances $\sigma^2$ and equal group sizes, the per-cell sample size for a one-shot design is

$$n_{\text{cell}} \;=\; \frac{2\,(z_{1-\alpha/2} + z_{1-\beta})^2\,\sigma^2}{\Delta^2} \;\approx\; \frac{15.68\,\sigma^2}{\Delta^2},$$

since $z_{1-\alpha/2} \approx 1.96$ and $z_{1-\beta} \approx 0.84$ so $(1.96 + 0.84)^2 \approx 7.84$ and $2 \times 7.84 \approx 15.68$.

### A. One-Shot Design (one elicitation per person)

| Per-round SD $\sigma$ | **Per-cell $n$** | **Total $N$ (4 cells)** |
|---|---|---|
| 5 | **85** | 340 |

### B. Fifteen Rounds per Person (pay all 15 rounds)

When each subject provides $m = 15$ paid WTPs, observations within subject are autocorrelated. Using the usual *design effect*

$$\text{DE} \;=\; 1 + (m-1)\rho \;=\; 1 + 14\,\rho,$$

the *effective* number of independent observations per subject is $m/\text{DE}$. A conservative adjustment divides the one-shot requirement by $m/\text{DE}$ to obtain subjects per cell.

**Results for $\sigma = 5$.**

| ICC $\rho$ | **Per-cell $n$ (subjects)** | **Total $N$ (4 cells)** |
|---|---|---|
| 0.3 | **30** | 120 |
| 0.4 | **38** | 152 |
| 0.5 | **46** | 184 |

**Planned sample size.** Using the conservative assumption $\rho = 0.5$, we target 46 participants per cell, for $4 \times 46 = 184$ participants in total.

## Simulations

To validate the analytical power calculations, we conducted Monte Carlo simulations replicating the experimental design (four treatment arms, 15 rounds per subject, and intra-subject correlation $\rho = 0.50$). In each of 1,000 replications, we generated subject-level random intercepts, applied the theoretical treatment shifts, and estimated the interaction effect using OLS with standard errors clustered at the participant level. The proportion of significant replications at $\alpha = 0.05$ yields an empirical power of 0.84, confirming that the analytical approximation provides a reliable prediction of the true design sensitivity.

## Appendix

## A EVSI (and WTP) for a 3-Signal Bundle under Quadratic/BSR

**Setup.** Let the state be $\theta \in \{0, 1\}$ with prior $p = \Pr(\theta = 1)$. A purchased bundle delivers $n = 3$ conditionally independent signals. Under the quadratic (Brier) scoring rule, truthful reporting equals the posterior $p_3$ after the bundle. For a risk–neutral agent, willingness to pay equals EVSI:

$$\mathrm{WTP}_3(p) = \mathrm{EVSI}_3(p) = 100\big[\mathbb{E}(p_3^2) - p^2\big] = 100\,Var(p_3).$$

**State-symmetric signal technology and diagnosticity.** Conditional on $\theta = 1$, a single draw yields

$$(\text{pos}, \text{neg}, \text{nd}) = (\alpha,\ \beta,\ 1 - \alpha - \beta), \qquad 0 < \beta < \alpha < 1.$$

Under $\theta = 0$ the informative probabilities swap: $(\text{pos}, \text{neg}) = (\beta,\ \alpha)$. Let $I \equiv \alpha + \beta$ denote the informative share and define the *diagnosticity (likelihood-ratio) parameter*

$$\lambda \ \equiv\ \frac{\alpha}{\beta}\ >\ 1.$$

**Distribution of evidence.** Let $K \sim \mathrm{Binom}(3, I)$ be the number of informative draws in the bundle, and define $D = \#\text{pos} - \#\text{neg} \in \{-k, -k + 2, \dots, k\}$ given $K = k$. By symmetry,

$$\Pr(D = d \mid K = k) = \binom{k}{\frac{k+d}{2}} 2^{-k}, \qquad d \equiv k \pmod 2.$$

**Posterior update.** Each pos contributes a Bayes factor $\lambda$ and each neg contributes $\lambda^{-1}$, so the net Bayes factor is $\lambda^D$. Hence, for prior $p$,

$$p_{k,d}(p) = \frac{p\,\lambda^d}{1 - p + p\,\lambda^d}.$$

**EVSI in closed form.** Let $p_3 = p_{K,D}(p)$. Then

$$\mathrm{EVSI}_3(p; I, \lambda) = 100 \left[ \sum_{k=0}^{3} \binom{3}{k} I^k (1-I)^{3-k} \sum_{d=-k,\ \text{step } 2}^{k} \binom{k}{\frac{k+d}{2}} 2^{-k} \big(p_{k,d}(p)\big)^2 \; - \; p^2 \right].$$

*Explanation.* The outer sum averages over the possible numbers of informative signals $K = k$ in the three-signal bundle (with $K \sim \mathrm{Binom}(3, I)$); the inner sum averages, given $k$, over the net balance $D = d$ of positive vs. negative signals. For each case $(k, d)$ we compute the posterior $p_{k,d}(p)$ and its quadratic score $(p_{k,d}(p))^2$, subtract the baseline score $p^2$ (no information), and finally multiply by 100 to express the EVSI in points.

## B "Narrative-boosted nondiagnostic" transformation

**Goal.** We model the narrative as making a fraction of nondiagnostic draws behave as *perceived* informative draws, while preserving the per-draw diagnosticity (likelihood ratio) $\lambda = \alpha/\beta$.

**Baseline.** Let $(\alpha, \beta, 1-\alpha-\beta)$ be the (perceived) probabilities of $(\mathrm{pos}, \mathrm{neg}, \mathrm{nd})$, with informative share $I \equiv \alpha + \beta$ and $\lambda \equiv \alpha/\beta > 1$.

**Transformation.** Choose $\phi \in [0, 1]$, the fraction of nondiagnostic mass $(1 - I)$ that becomes informative. The transformed informative share is

$$I' \; = \; I + \phi(1 - I), \qquad \text{and hence } \mathrm{nd}' \; = \; 1 - I' \; = \; (1 - I)(1 - \phi).$$

To keep $\lambda$ unchanged, split the added informative mass proportionally to $\alpha$ and $\beta$:

$$\alpha' \; = \; \alpha + \phi(1-I)\frac{\alpha}{I} \; = \; \frac{\lambda}{1+\lambda} I', \qquad \beta' \; = \; \beta + \phi(1-I)\frac{\beta}{I} \; = \; \frac{1}{1+\lambda} I'.$$

**Example (halving nondiagnostic).** If the baseline is $(\alpha, \beta, \mathrm{nd}) = (0.45, 0.15, 0.40)$ ($I = 0.60$, $\lambda = 3$) and we "halve" nondiagnostic, then $\phi = 0.5$:

$$I' = 0.60 + 0.5 \times 0.40 = 0.80, \quad (\alpha', \beta', \mathrm{nd}') = \Big(0.60,\ 0.20,\ 0.20\Big).$$