1 Framework

In this paper, an alternative consists of a product and/or money. In the spirit of a multi-attribute choice setting, we represent each alternative by a collection of a quality attribute value and a money attribute value. A quality value can be interpreted as a subjective value of a product. A money value can be interpreted as the remaining dollar amount after purchasing the product.[1] Note that all attribute values are expressed in monetary terms. Formally, alternative j is denoted by $j = (q_j, m_j)$ in which $q_j \geqslant 0$ is a quality value in monetary terms and $m_j \geqslant 0$ is a money value. When an alternative is simply choosing money, we assume that the quality value is zero. Let A be a menu collecting alternatives available to a decision maker. For example, $A = \{j, \dots, j'\}$ means that alternatives $j, \dots, j'$ are available to a decision maker.

A multi-attribute choice model shows how an alternative is evaluated given a menu and predicts that an alternative that has the highest evaluation in the menu would be chosen. Each multi-attribute choice model has a different way of combining attribute values when evaluating an alternative. An evaluation of an alternative may depend on other attribute values in a menu.

In the following subsections, several multi-attribute choice models are introduced. The first one is a value maximization model, which is a benchmark model. The other models are range normalization models and a pairwise normalization model, which are the main models of interest.

1.1 Value maximization

A value maximization model (VM) evaluates an alternative by summing all the attribute values of the alternative. Formally, an evaluation of alternative j is:

$$V_{VM}(j) = q_j + m_j.$$

[1]Note that this is one way to interpret money values. In the experiment, an alternative is expressed as a bundle of a product and/or money. Prices are never mentioned during the experiment. This is because some multi-attribute choice models provide different predictions depending on how a decision maker encodes the attribute.

1.2 Range normalization

A range normalization model (RN) evaluates an alternative by summing the normalized attribute values where each normalization depends on the range of the associated attribute. Formally, an evaluation of alternative j in menu A is:

$$V_{RN}(j|A) = q_j(\max_{k\in A} q_k - \min_{k\in A} q_k)^\gamma + m_j(\max_{k\in A} m_k - \min_{k\in A} m_k)^\gamma,$$

where $\gamma > -1$. Note that the implications of a range normalization model differ depending on a value of γ. Specifically, if $\gamma < 0$, then a weight through normalization decreases as a range gets larger [Bushong et al., 2021]. If $\gamma > 0$, then a weight through normalization increases as a range gets larger [Kőszegi and Szeidl, 2013]. In this paper, we separate a range normalization model with a negative γ (RNn) and a range normalization model with a positive γ (RNp).

1.3 Pairwise normalization

A pairwise normalization model (PN) evaluates an alternative by summing normalized attribute values. In this model, the normalization works by pairwisely comparing an attribute value of the evaluated alternative and an attribute value of another alternative in the menu [Landry and Webb, 2021]. Formally, an evaluation of alternative j in menu A is:

$$V_{PN}(j|A) = \sum_{k\in A\setminus\{j\}} \frac{q_k}{\sigma + q_j + q_k} + \sum_{k\in A\setminus\{j\}} \frac{m_j}{\sigma + m_j + m_k},$$

where $\sigma > 0$ represents a “semi-saturation” constant.

2 Experimental Design

The experiment is designed to test the multi-attribute choice models introduced above. To this end, the experiment is composed of two parts: (i) a quality elicitation step (Task 1) and (ii) a main choice task (Task 2). In the quality elicitation step, subjective quality values of products are elicited. Using this information, we generate menus that can distinguish the models. Detailed explanations of each part are described below.

Table 1: List of snacks

#	Product Description	Size	Category
1	Cheetos Crunchy, 8.5 oz (240.9g)	Large	Chips 1
2	Doritos Nacho Cheese, 9.25 oz (262.2g)	Large	Chips 1
3	Fritos Original, 9.25 oz (262.2g)	Large	Chips 1
4	Lay's Classic, 8 oz (226.8g)	Large	Chips 2
5	Pringles Original, 5.2 oz (149g)	Large	Chips 2
6	Ruffles Original, 8.5 oz (240.9g)	Large	Chips 2
7	Cheezit Original, 7 oz (198g)	Large	Crackers
8	Goldfish Cheddar, 6.6 oz (187g)	Large	Crackers
9	Ritz Original, 10.3 oz (291g)	Large	Crackers
10	Chips Ahoy Original, 13 oz (368g, 33 cookies)	Large	Cookies
11	Oreo, 13.29 oz (376g, 34 cookies)	Large	Cookies
12	Milano Dark Chocolate, 6 oz (170g, 15 cookies)	Large	Cookies
13	Cheetos Crunchy, 1 oz (28.3g)	Small	
14	Doritos Nacho Cheese, 1 oz (28.3g)	Small	
15	Fritos Original, 1 oz (28.3g)	Small	
16	Lay's Classic, 1 oz (28.3g)	Small	
17	Pringles Original, 2.3 oz (67g)	Small	
18	Ruffles Original, 1 oz (28.3g)	Small	
19	Cheezit Original, 1 oz (28.3g)	Small	
20	Goldfish Cheddar, 1 oz (28.3g)	Small	
21	Ritz Original, 3.4 oz (96g)	Small	
22	Chips Ahoy Original, 1.55 oz (44g ,4 cookies)	Small	
23	Oreo, 2.4 oz (68g, 6 cookies)	Small	
24	Milano Dark Chocolate, 0.75 oz (21g, 2 cookies)	Small	

2.1 Quality evaluation task

In the quality elicitation task, we elicit subjective quality values of 24 consumer products using a multiple price list method. Consumer products used in the experiment are snacks that are familiar to student subjects. A full list of snacks is presented in Table 1.

In each round, a picture, a name, and a weight of a product appear on a screen as presented in Figure 1. Then a subject is asked whether they prefer the product (Option A) to an amount of money (Option B). This binary question is asked multiple times by varying the amount of money in Option B ranging from \$0.01 to \$8 in increments of \$0.01. Instead of asking a subject to make 800 choices in each round,

Figure 1: A screenshot of a decision page in the evaluation task

Round 1 of Task 1

The product is **Pringles Original, 5.2 oz (149g)**.

Question #		**Option A**		**Option B**
1	Would you rather have:	Product	or	$0.01
2	Would you rather have:	Product	or	$0.02
⋮	⋮	⋮	⋮	⋮
799	Would you rather have:	Product	or	$7.99
800	Would you rather have:	Product	or	$8.00

At which dollar value ($) would you switch?

Note: The value will be automatically rounded to two decimal places.

Submit

we ask them at which dollar value they would like to switch from Option A to Option B. Once they report the switch point, then Option A is chosen for the first question to the question right before the switch point, and Option B is chosen for all questions after the switch point. We say that a dollar value at the switch point is a subjective quality value of a product.[2] The order of rounds is randomized at the subject level.

2.2 Main choice task

A decision problem of the main choice task is to choose the most preferred alternative given a menu. Every menu in the main choice task consists of three alternatives. Specifically, an alternative with a high-quality product is denoted by

[2] Im [2023] theoretically shows that if an injective assumption is satisfied, then a dollar value at the switch point is equal to a subjective quality value of a product. The range normalization models and the pairwise normalization model studied in this paper satisfy the injective assumption.

Table 2: Datasets

		$m_o = 8$	$m_o = 10$
Vertically differentiated	Pair 1	Dataset V1L	Dataset V1H
	Pair 2	Dataset V2L	Dataset V2H
Horizontally differentiated	Pair 1	Dataset H1L	Dataset H1H
	Pair 2	Dataset H2L	Dataset H2H

$h = (q_h, m_h)$, an alternative with a low-quality product is denoted by $\ell = (q_\ell, m_\ell)$, and an outside option is denoted by $o = (0, m_o)$ in which $q_h > q_\ell > 0$ and $m_o > \max\{m_h, m_\ell\} \geqslant \min\{m_h, m_\ell\} > 0$. A menu in the main choice task is denoted by $A = \{h, \ell, o\}$.

Before the main choice task, four pairs of high-quality and low-quality products are selected using the elicited quality information from the elicitation task. More specifically, as presented in Table 1, we have 12 kinds of snacks with a large size and a small size. We say that a pair of products is *vertically differentiated* when the two snacks are the same but have different sizes. Thus, we have 12 vertically differentiated product pairs. As presented in Column 4 in Table 1, 12 large snacks can be classified into 4 different categories.[3] We say that a pair of products is *horizontally differentiated* when the two snacks are different but in the same category. Thus, we have 12 horizontally differentiated product pairs.

We select two pairs of vertically differentiated products and two pairs of horizontally differentiated products by the following criteria. First, for each product, we check whether the elicited quality value is greater than or equal to \$0.1 and smaller than or equal to \$6. Second, for each product pair, we check whether the difference between the two elicited quality values is greater than or equal to \$0.5. Among the pairs that pass the criteria, we randomly select two vertically differentiated product pairs and two horizontally differentiated product pairs. If not enough product pairs pass the criteria, then we first collect all the product pairs that pass the criteria, if they exist. Then we randomly select any product pairs to match four pairs. However, in the main analysis, we exclude all the product pairs that do not pass the criteria.

[3]As described later, we select two pairs of horizontally differentiated products and use them in the main choice task. When selecting the product pairs, we need a large enough quality gap between the two products. Since we do not expect a large quality gap between small-size products, we only consider large-size products for the horizontally differentiated product pairs.

Table 3: Types according to multi-attribute choice models

Model	Parameter Region	Type
RNn	$-1 < \gamma < -0.5$	$\tau_1^{RNn} = hohohh$
	$-0.5 < \gamma < -0.1$	$\tau_2^{RNn} = oohohh$
	$-0.1 < \gamma < 0$	$\tau_3^{RNn} = oooohh$
RNp	$0 < \gamma < 0.1$	$\tau_1^{RNp} = oooohh$
	$0.1 < \gamma < 0.5$	$\tau_2^{RNp} = ooooh$
	$0.5 < \gamma$	$\tau_3^{RNp} = oooooo$
PN	$0 < \sigma < 50$	$\tau_1^{PN} = hhhhhh$
	$50 < \sigma < 200$	$\tau_2^{PN} = oohhhh$
	$200 < \sigma$	$\tau_3^{PN} = oooohh$

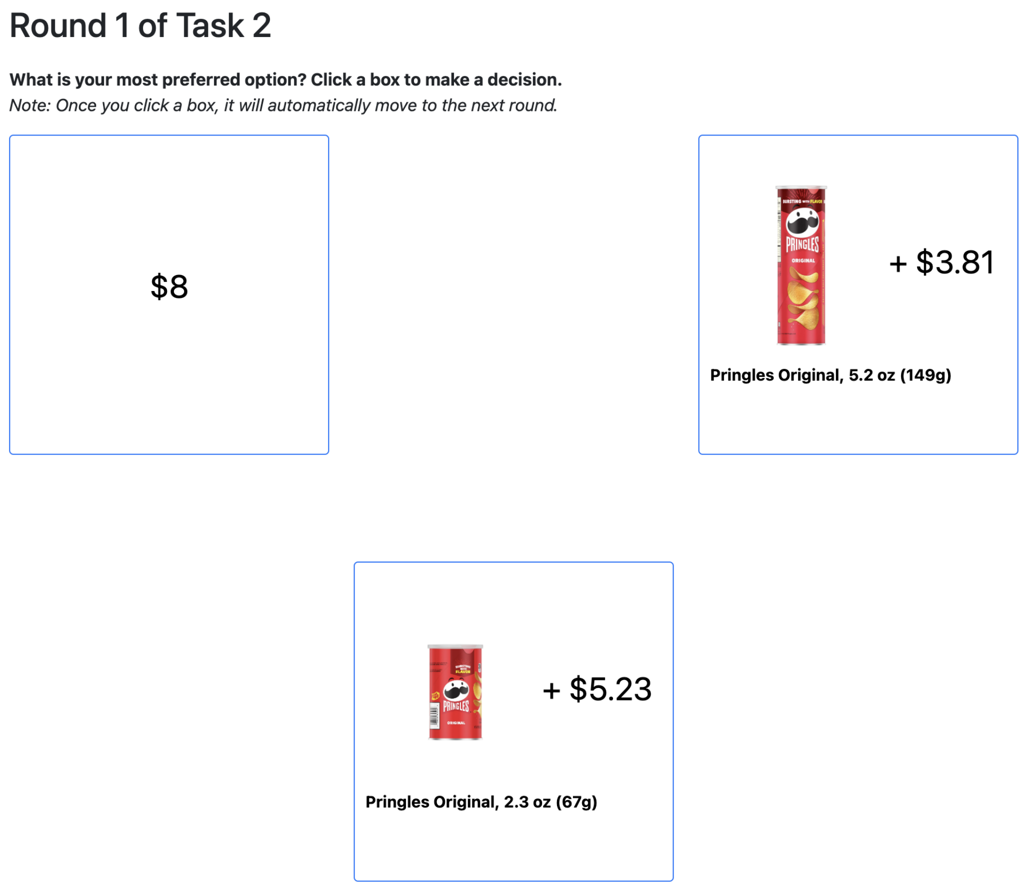
We also have two different outside options, either $m_o = 8$ or $m_o = 10$. Hence, we have 8 combinations depending on a product pair and an outside option in the main choice task as summarized in Table 2. For each combination, we generate 6 menus that distinguish the models by allowing different choice types.

Here we briefly discuss which choice types are allowed in each model. Suppose that alternative j_k is chosen in menu A_k. Then a *choice pattern* is a set of six alternatives denoted by $j_1j_2j_3j_4j_5j_6$ where the k-th alternative indicates a choice in menu A_k. Let $\tau_t^M = j_1j_2j_3j_4j_5j_6$ be the t-th *type* from model M where the choice pattern is predicted by model M. For each combination in Table 2, 6 menus are targeted to generate $\tau_1^{RNn} = hohohh$, $\tau_2^{RNn} = oohohh$, and $\tau_3^{RNn} = oooohh$ according to a range normalization model with a negative γ (RNn), $\tau_1^{RNp} = oooohh$, $\tau_2^{RNp} oooooh$, and $\tau_3^{RNp} oooooo$ according to a range normalization model with a positive γ (RNp), and $\tau_1^{PN} = hhhhhh$, $\tau_2^{PN} = oohhhh$, and $\tau_3^{PN} = oooohh$ according to a pairwise normalization model (PN). The targeted types for each model are summarized in Table 3.

For instance, consider the range normalization model. If $-1 < \gamma < -0.5$, then the model predicts that h would be chosen in menus A_1, A_3, A_5, and A_6 and o would be chosen in menus A_2 and A_4. If $-0.5 < \gamma < -0.1$, then the model predicts *oohohh*. If $-0.1 < \gamma < -0$, then the model predicts *oooohh*.

Note that the six menus depend on the quality values of a product pair and the outside option. For example, given $q_h = 4$, $q_\ell = 1$, and $m_o = 8$, we generate $A_1 = \{(4, 3.38), (1, 2.67), (0, 8)\}$, $A_2 = \{(4, 3.36), (1, 3.36), (0, 8)\}$,

Figure 2: A screenshot of decision page in Task 2



$A_3 = \{(4, 3.84), (1, 2.01), (0, 8)\}$, $A_4 = \{(4, 3.82), (1, 3.82), (0, 8)\}$, $A_5 = \{(4, 4.16), (1, 2), (0, 8)\}$, and $A_6 = \{(4, 4.73), (1, 2), (0, 8)\}$.

To sum up, each subject makes 48 choices (6 menus for 8 combinations) in the main choice task. In each round, alternatives from a menu are displayed as in Figure 2. A subject can choose the most preferred alternative by clicking a box. The order of alternatives is randomized at the subject level. The order of rounds is randomized at the subject level as well.

3 Analysis Plan

In this section, we discuss three comparative statics we can do from the current design. Based on the discussed comparative statics, we ran simulations and determined the sample size to be 60. Moreover, by running further simulations, we checked that the sample size of 60 is enough to do a model selection exercise.

In the main analysis, we consider data within the same product type, i.e., either vertically differentiated products or horizontally differentiated products. In this section, we focus on datasets using vertically differentiated products. This means that when we aggregate data, each subject has 4 choice patterns or 24 choices in total.

3.1 Determine the sample size

Let $\mu^M \in \Delta$ be a distribution over the type space of model M in which $\mu_t^M \in [0,1]$ is a fraction of type τ_t^M and $\sum_{t=1}^3 \mu_t^M = 1$. We assume that a decision maker makes a choice consistent with a model with a probability of $1-\varepsilon$ and makes a random choice with a probability of ε where $\varepsilon \in [0,1]$. For instance, consider a pairwise normalization model with $\sigma = 20$. Given menu A_1, the model predicts that h would be chosen (see Table **??**). Now, suppose that a decision maker makes a choice according to the PN model with a probability of $1-\varepsilon$ and makes a random choice with a probability of ε. Then the decision maker would choose h with a probability of $(1-\varepsilon) + \frac{1}{3}\varepsilon = 1 - \frac{2}{3}\varepsilon$, choose ℓ with a probability of $\frac{1}{3}\varepsilon$, and choose o with a probability of $\frac{1}{3}\varepsilon$. Note that if $\varepsilon = 0$, then it becomes a deterministic case. If $\varepsilon = 1$, then the decision maker makes completely random choices. We have a stochastic model when $\varepsilon > 0$.

Assuming the above error structure, we can consider the following comparative statics. The first proposition shows how choice probabilities change by comparing menus A_3 and A_4. The second proposition compares choice probabilities between menus A_2 and A_4. The third proposition compares choice probabilities between menus A_5 and A_6.

Proposition 1. *Consider menus A_3 and A_4. Let an error rate of $\varepsilon \in [0,1)$ be given.*

1. *If a decision maker chooses according to the RNn model, then the choice probability of h increases, the choice probability of ℓ does not change, and the choice probability of o decreases as the menu changes from A_4 to A_3.*

2. *If a decision maker chooses according to the RNp model, then the choice probabilities of all alternatives are the same between the menus.*

3. *If a decision maker chooses according to the PN model, then the choice probabilities of all alternatives are the same between the menus.*

Proof. Let $\Pr^M(j|A)$ be a choice probability of alternative j in menu A according to model M. If a decision maker chooses an alternative according to the RNn model, then the probabilities of choosing h are $\Pr^{RNn}(h|A_3) = (\mu_1^{RNn} + \mu_2^{RNn})(1 - \frac{2}{3}\varepsilon) + \mu_3^{RNn}\frac{1}{3}\varepsilon > \frac{1}{3}\varepsilon = \Pr^{RNn}(h|A_4)$, the probabilities of choosing ℓ are $\Pr^{RNn}(\ell|A_3) = \frac{1}{3}\varepsilon = \Pr^{RNn}(\ell|A_4)$, and the probabilities of choosing o are $\Pr^{RNn}(o|A_3) = (\mu_1^{RNn} + \mu_2^{RNn})\frac{1}{3}\varepsilon + \mu_3^{RNn}(1 - \frac{2}{3}\varepsilon) < 1 - \frac{2}{3}\varepsilon = \Pr^{RNn}(o|A_2)$.

If a decision maker chooses an alternative according to the RNp model, then the probabilities of choosing h are $\Pr^{RNp}(h|A_3) = \frac{1}{3}\varepsilon = \Pr^{RNp}(h|A_4)$, the probabilities of choosing ℓ are $\Pr^{RNp}(\ell|A_3) = \frac{1}{3}\varepsilon = \Pr^{RNp}(\ell|A_4)$, and the probabilities of choosing o are $\Pr^{RNp}(o|A_3) = 1 - \frac{2}{3}\varepsilon = \Pr^{RNp}(o|A_4)$.

If a decision maker chooses an alternative according to the PN model, then the probabilities of choosing h are $\Pr^{PN}(h|A_3) = (\mu_1^{PN} + \mu_2^{PN})(1 - \frac{2}{3}\varepsilon) + \mu_3^{PN}\frac{1}{3}\varepsilon = \Pr^{PN}(h|A_4)$, the probabilities of choosing ℓ are $\Pr^{PN}(\ell|A_3) = \frac{1}{3}\varepsilon = \Pr^{PN}(\ell|A_4)$, and the probabilities of choosing o are $\Pr^{PN}(o|A_3) = (\mu_1^{PN} + \mu_2^{PN})\frac{1}{3}\varepsilon + \mu_3^{PN}(1 - \frac{2}{3}\varepsilon) = \Pr^{PN}(o|A_4)$. □

Proposition 2. *Consider menus A_5 and A_6. Let an error rate of $\varepsilon \in [0, 1)$ be given.*

1. *If a decision maker chooses according to the RNn model, then the choice probabilities of all alternatives are the same between the menus.*

2. *If a decision maker chooses according to the RNp model, then the choice probability of h increases, the choice probability of ℓ does not change, and the choice probability of o decreases as the menu changes from A_5 to A_6.*

3. *If a decision maker chooses according to the PN model, then the choice probabilities of all alternatives are the same between the menus.*

Proof. If a decision maker chooses an alternative according to the RNn model, then the probabilities of choosing h are $\Pr^{RNn}(h|A_5) = 1 - \frac{2}{3}\varepsilon = \Pr^{RNn}(h|A_6)$, the probabilities of choosing ℓ are $\Pr^{RNn}(\ell|A_5) = \frac{1}{3}\varepsilon = \Pr^{RNn}(\ell|A_6)$, and the probabilities of choosing o are $\Pr^{RNn}(o|A_5) = \frac{1}{3}\varepsilon = \Pr^{RNn}(o|A_6)$.

If a decision maker chooses an alternative according to the RNp model, then the probabilities of choosing h are $\Pr^{RNp}(h|A_6) = (\mu_1^{RNp} + \mu_2^{RNp})(1 - \frac{2}{3}\varepsilon) + \mu_3^{RNp}\frac{1}{3}\varepsilon > \mu_1^{RNp}(1 - \frac{2}{3}\varepsilon) + (\mu_2^{RNp} + \mu_3^{RNp})\frac{1}{3}\varepsilon = \Pr^{RNp}(h|A_5)$, the probabilities of choosing ℓ are $\Pr^{RNp}(\ell|A_6) = \frac{1}{3}\varepsilon = \Pr^{RNp}(\ell|A_5)$, and the probabilities of choosing o are $\Pr^{RNp}(o|A_6) = (\mu_1^{RNp} + \mu_2^{RNp})\frac{1}{3}\varepsilon + \mu_3^{RNp}(1 - \frac{2}{3}\varepsilon) < \mu_1^{RNp}\frac{1}{3}\varepsilon + (\mu_2^{RNp} + \mu_3^{RNp})(1 - \frac{2}{3}\varepsilon) = \Pr^{RNp}(o|A_5)$.

If a decision maker chooses an alternative according to the PN model, then the probabilities of choosing h are $\Pr^{PN}(h|A_5) = 1 - \frac{2}{3}\varepsilon = \Pr^{PN}(h|A_6)$, the probabilities of choosing ℓ are $\Pr^{PN}(\ell|A_5) = \frac{1}{3}\varepsilon = \Pr^{PN}(\ell|A_6)$, and the probabilities of choosing o are $\Pr^{PN}(o|A_5) = \frac{1}{3}\varepsilon = \Pr^{PN}(o|A_6)$. □

Proposition 3. *Consider menus A_2 and A_4. Let an error rate of $\varepsilon \in [0, 1)$ be given.*

1. *If a decision maker chooses according to the RNn model, then the choice probabilities of all alternatives are the same between the menus.*

2. *If a decision maker chooses according to the RNp model, then the choice probabilities of all alternatives are the same between the menus.*

3. *If a decision maker chooses according to the PN model, then the choice probability of h increases, the choice probability of ℓ does not change, and the choice probability of o decreases as the menu changes from A_2 to A_4.*

Proof. If a decision maker chooses an alternative according to the RNn model, then the probabilities of choosing h are $\Pr^{RNn}(h|A_4) = \frac{1}{3}\varepsilon = \Pr^{RNn}(h|A_2)$, the probabilities of choosing ℓ are $\Pr^{RNn}(\ell|A_4) = \frac{1}{3}\varepsilon = \Pr^{RNn}(\ell|A_2)$, and the probabilities of choosing o are $\Pr^{RNn}(o|A_4) = 1 - \frac{2}{3}\varepsilon = \Pr^{RNn}(o|A_2)$.

If a decision maker chooses an alternative according to the RNp model, then the probabilities of choosing h are $\Pr^{RNp}(h|A_4) = \frac{1}{3}\varepsilon = \Pr^{RNp}(h|A_2)$, the probabilities of choosing ℓ are $\Pr^{RNp}(\ell|A_4) = \frac{1}{3}\varepsilon = \Pr^{RNp}(\ell|A_2)$, and the probabilities of choosing o are $\Pr^{RNp}(o|A_4) = 1 - \frac{2}{3}\varepsilon = \Pr^{RNp}(o|A_2)$.

If a decision maker chooses an alternative according to the PN model, then the probabilities of choosing h are $\Pr^{PN}(h|A_4) = (\mu_1^{PN} + \mu_2^{PN})(1 - \frac{2}{3}\varepsilon) + \mu_3^{PN}\frac{1}{3}\varepsilon > \mu_1^{PN}(1 - \frac{2}{3}\varepsilon) + (\mu_2^{PN} + \mu_3^{PN})\frac{1}{3}\varepsilon = \Pr^{PN}(h|A_2)$, the probabilities of choosing ℓ are

$\Pr^{PN}(\ell|A_4) = \frac{1}{3}\varepsilon = \Pr^{PN}(\ell|A_2)$, and the probabilities of choosing o are $\Pr^{PN}(o|A_4) = (\mu_1^{PN} + \mu_2^{PN})\frac{1}{3}\varepsilon + \mu_3^{PN}(1 - \frac{2}{3}\varepsilon) < \mu_1^{PN}\frac{1}{3}\varepsilon + (\mu_2^{PN} + \mu_3^{PN})(1 - \frac{2}{3}\varepsilon) = \Pr^{PN}(o|A_2)$. $\square$

To determine the sample size, we do the following power analysis. First, consider Proposition 1 and the table below:

	Choice distribution 1	Choice distribution 2
h	$\Pr(h\|A_3)$	$\Pr(h\|A_4)$
ℓ	$\Pr(\ell\|A_3)$	$\Pr(\ell\|A_4)$
o	$\Pr(o\|A_3)$	$\Pr(o\|A_4)$

Proposition 1 implies that if the RNn model is true, then a distribution over choice probabilities in menu A_3 (i.e., choice distribution 1) and a distribution over choice probabilities in menu A_4 (i.e., choice distribution 2) would be significantly different. If the RNp or the PN models are true, then the choice distributions are not significantly different. To do the power analysis, we run simulations assuming that the RNn model is true and generate choice data. Then we count how many times the chi-square test rejects the null hypothesis at the significance level of 0.05.

Specifically, suppose that the sample size is N. For each subject, we randomly assign one of the RNn types according to $\mu^{RNn} = (0.15, 0.35, 0.5)$, an error rate according to $\varepsilon \sim U[0.05, 0.25]$, and two pairs of quality values that satisfy the criteria.[4] Given quality values, we generate 6 menus for each combination in Table 2. As we focus on vertically differentiated products in this section, we have 24 menus in total following the current experimental design. Next, based on the assigned parameter values, choice data are generated. By aggregating the choices from the four datasets, we obtain choice distributions and conduct the chi-square test. We repeat this simulation 100 times.

Row (1) in Table 4 shows the percentage of rejecting the null with different sample sizes. Note that for the chi-square test to be valid, the expected frequency for each cell should be greater than 5. When computing the power, we exclude invalid cases and separately count them below. For example, the power of 1 when the sample size is 10 means that the null hypothesis is rejected 1 time out of 1 valid case. The number of invalid cases approaches zero as the sample size increases. In particular, we have zero invalid cases when the sample size is greater than or equal to 50.

[4]The type distribution, the error distribution, and the quality data are based on the pilot.

Table 4: Power Analysis

	DGP		10	20	30	40	50	60	70	80
(1)	RNn	Power	1	1	1	1	1	1	1	1
		Invalid	99	89	28	10	0	0	0	0
(2)	RNp	Power	·	0.52	0.52	0.65	0.75	0.81	0.9	0.88
		Invalid	100	79	37	5	0	0	0	0
(3)	PN	Power	·	0.52	0.72	0.80	0.86	0.87	0.96	0.97
		Invalid	100	79	32	6	2	0	0	0

Next, Proposition 2 implies that if the RNp model is true, then the choice distributions between menus A_5 and A_6 are significantly different. If the RNn or the PN models are true, then choice distributions are not significantly different. We run similar simulations 100 times assuming that the RNp model with $\mu^{RNp} = (0.7, 0.15, 0.15)$ and $\varepsilon \sim U[0.05, 0.25]$ is true.[5] Row (2) in Table 4 shows that we achieve a power greater than 0.8 with no invalid case when the sample size is 60. In fact, when the sample size is greater than 55, the power is roughly 0.8 with 1 or 0 invalid cases.

Lastly, Proposition 3 implies that if the PN model is true, then the choice distributions between menus A_2 and A_4 are significantly different. If the RNn or the RNp models are true, then choice distributions are not significantly different. Again, we run simulations 100 times assuming that the PN model with $\mu^{PN} = (0.05, 0.15, 0.8)$ and $\varepsilon \sim U[0.05, 0.25]$ is true.[6] Row (3) in Table 4 shows that when the sample size is 40, we achieve a power of 0.8 with 6 invalid cases. The power increases and the number of invalid cases decreases as the sample size increases. In particular, the number of invalid cases becomes zero when the sample size becomes 60.

Overall, when the sample size is 55, we achieve powers greater than or roughly equal to 0.8 with 1 or 0 invalid cases in all three simulations. Also, considering that around 10% of subjects fail to pass the product selection criteria, we decided to have a sample size of 60 for this experiment.

3.2 Model selection exercise

We use a cross-validation method to do model selection exercises. The main purpose of this subsection is to check whether the sample size of 60 is enough to pick

[5]The type distribution is based on the estimation result in Somerville [2022].

[6]The type distribution and the error distribution are based on the pilot.

a true winning model by running simulations.

First, we construct a likelihood based on types and choice patterns in the spirit of Harless and Camerer [1994]. To illustrate, consider the PN model. Remember that for each dataset, the PN model allows three types, i.e., *hhhhhh*, *oohhhh*, and *ooooh h* (see Table **??**). If we observe a choice pattern of *hhhhhh* from a dataset, then type *hhhhhh* interprets that the decision maker makes no mistakes throughout the six menus. The likelihood of this is $\mu_1^{PN}(1-\frac{2}{3}\varepsilon)^6$. Type *oohhhh* interprets the observed choice pattern by saying that mistakes happened in menus A_1 and A_2. The likelihood of this is $\mu_2^{PN}(\frac{1}{3}\varepsilon)^2(1-\frac{2}{3}\varepsilon)^4$. Lastly, type *oooohh* interprets the observed choice pattern by saying that mistakes happened in menus A_1, A_2, A_3, and A_4. The likelihood of this is $\mu_3^{PN}(\frac{1}{3}\varepsilon)^4(1-\frac{2}{3}\varepsilon)^2$.

To summarize, given type t and an observed choice pattern, let C be the number of alternatives that are consistent with type t and the observed choice pattern. Then the likelihood is $\mu_t(\frac{1}{3}\varepsilon)^{6-C}(1-\frac{2}{3}\varepsilon)^C$. Therefore, the likelihood of the entire observed choice patterns based on the PN model is:

$$
\begin{aligned}
Likelihood^{PN} = & \left[\mu_1^{PN}(1-\frac{2}{3}\varepsilon)^6 + \mu_2^{PN}(\frac{1}{3}\varepsilon)^2(1-\frac{2}{3}\varepsilon)^4 + \mu_3^{PN}(\frac{1}{3}\varepsilon)^4(1-\frac{2}{3}\varepsilon)^2\right]^{\Pr(hhhhhh)} \\
& \times \ldots \\
& \times \left[\mu_1^{PN}(\frac{1}{3}\varepsilon)^6 + \mu_2^{PN}(\frac{1}{3}\varepsilon)^4(1-\frac{2}{3}\varepsilon)^2 + \mu_3^{PN}(\frac{1}{3}\varepsilon)^2(1-\frac{2}{3}\varepsilon)^4\right]^{\Pr(oooooo)}
\end{aligned}
$$

Similarly, let $Likelihood^{RNn}$ be the likelihood of the entire observed choice patterns based on the RNn model and $Likelihood^{RNp}$ be the likelihood of the entire observed choice patterns based on the RNp model.

Note that we have 4 datasets from vertically differentiated products where each dataset has a choice pattern consisting of 6 menus. Hence, we do 4-fold cross-validation. This means that we train a model using 3 datasets, i.e., estimate a type distribution and an error rate by maximizing a corresponding likelihood defined above. Given the estimated type distribution and error rate, we compute an out-of-sample likelihood using the remaining dataset. We repeat this 4 times by alternating the train datasets and a test dataset. Then we average 4 out-of-sample likelihoods. We repeat this for each model. Finally, we select a winner model by finding the one that has the largest average out-of-sample likelihood.

The purpose of this subsection is to check whether the sample size of 60 is enough to pick a true model by the cross-validation exercise described above. To this end, we generate data following the same steps as explained in Section 3.1. When the RNn model is true, i.e., the data is generated by the RNn model, then the RNn model wins 100 out of 100 times. When the RNp is the true model, the RNp model wins 100 out of 100 times. When the PN is the true model, the PN model wins 100 out of 100 times. Therefore, we confirm that the sample size of 60 is enough to pick the true model through the cross-validation exercise.

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