Pre-Analysis Plan for "Wheels of change: Impact of bicycles on female education and empowerment in Zambia"

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1. Introduction

This document outlines the pre-analysis plan for "Wheels of change: Impact of bicycles on female education and empowerment in Zambia" study. The goal of this document is to pre-specify our planned methodology, hypothesis and specifications for analyzing the impacts of a program delivered to adolescent girls, discussed in detail below. In addition to the ex-ante planned analysis specified here, we may conduct additional exploratory analyses as these are nevertheless an important means to discovery. However, this document serves as a record of our planned analyses and all analyses not planned ex-ante will be marked as such when reporting results.

2. Motivation for the study

While many countries have made significant progress towards gender equality in education, labor force participation, and political representation in recent decades, discriminatory social norms that limit girls' access to education and labor force participation remains a challenge in many parts of the world. In Zambia, gender parity in primary schooling has improved considerably, however there continues to be significant gaps in school enrollment, attendance, and dropouts with a significant increase in adolescent years. Exhaustion, insecurity and lack of

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support and incentives discourage many students walking long distances to come to school.

One of the key reasons hypothesized for low enrollment, attendance and dropouts is the long distance to school. In particular, this is a critical barrier for girls in rural Zambia, especially around the time when girls start menstruating. According to ZEDS (2002), the second most cited reason for late enrollment in school is distance, with the problem being more severe in rural areas compared to urban areas. For example, 42% of children in Southern province started school late due to distance compared to 16% in Copperbelt province.

According to the Zambia DHS 2007, 10.3% of all girls 9-13 were out of school in Zambia. Identifying cost-effective and scalable strategies that directly address gender gap in school attendance, which is likely to be correlated with learning outcomes among girls, is therefore of considerable policy interest in Zambia. One such policy that addresses this twin problem is providing bicycles to girls to help reduce the distance cost of schooling and at the same time improve safety. Recent research by Muralidharan and Prakash (2017) shows that a similar policy in India was very successful in reducing the gender gap in enrollment. This policy increased girls' enrollment in secondary schools by 32% and reduced the gender gap by 40%. They also find an 18% increase in the number of girls who appear for the high-stakes secondary school certificate exam and a 12% increase in the number of girls who pass the exam.

Implementing a bicycle program in Zambia has the potential to address directly the first order problems of *school enrollment*, *attendance* and *dropouts* among girls in rural areas. In addition, the benefit of female education, and the fact that a transfer that improves mobility is being made to girls who generally receive little support from their family is likely to have an impact on other important outcomes like mobility, aspiration, and empowerment. In particular, empowerment of girls is one of the top priorities of the UN's Sustainable Development Goals (Goal 5). Traditionally, outcomes related to female empowerment are measured by how much bargaining power (married) women have within the household. In this study, we aim to measure how the transfer of a valuable asset to an adolescent girl can change her bargaining power within her household, which would affect outcomes that matter not just in the short run but could have a long-lasting impact.

In this study, we conduct an experiment to estimate the causal impact of a conditional kind

transfer (CKT) program that provides a bicycle to a school-going girl child of a household that generally provides no or very little support in rural Zambia.

Apart from the first order effects on schooling outcomes, it is important for countries to improve female education from a macro perspective, as this will directly contribute to the economic growth through both (a) the direct channel of improved human capital of female participants in the labor force, and (b) the indirect channel of improved human capital of the next generation. For instance, several studies have shown high-levels of inter-generational transmission of human capital from mothers to children, e.g. (Currie and Moretti, 2003).

This study directly contributes to existing research on the most commonly used demand-side policy intervention, i.e. conditional cash transfers (CCTs), to increase female schooling in developing countries. In particular, identifying cost-effective and scalable ways to improve female education is of considerable policy interest especially for developing countries. Furthermore, this intervention makes two unique contributions to this literature. First, we believe this is the first experimental study that estimates the impact of a large CKT (as opposed to CCT) program on girls' education. Second, we are not aware of any study in the CCT literature that targets transfer to adolescent girls themselves. Third, results from this study can help to address the broader debate on *cash* versus *kind* transfers as tools for social policy in the developing world.

Overall, this study will not only provide a clear understanding of the impact of reducing the distance cost to school for girl, but also address the larger question of whether or not such an asset transfer has an impact on girls' empowerment, a question of immense policy interest for both developed and the developing world.

3. Study Overview

This study is a randomized controlled trial implemented in the Southern province of Zambia. The goal is to examine the impact of providing bicycles to school-going adolescent girls on education and empowerment outcomes.

a. Study design

The experiment was designed as a multi-treatment design with schools randomly allocated to one of two treatment arms or control.

Treatment 1(T1): Payment Arm - 25 schools

The first treatment is the same as the BEEP (Bicycle Education and Empowerment Program) intervention that WBR (World Bicycle Relief) has rolled out in other parts of Zambia (in 19 districts since 2009). Students received a bicycle on the condition that the bicycle be used primarily to travel to school. A Bicycle Supervisory Committee (BSC) was formed consisting mostly of teachers, PTA members or local leaders, which is in charge of monitoring the program. A field mechanic was trained for each school, who provided maintenance checks and repairs for a fee borne by the recipient of the bicycle.

Each school was then provided with a startup spare parts kit and each beneficiary student was required to pay a **contribution of 50 Kwacha (~ $5)** toward this kit.

Treatment 2 (T2): No Payment Arm - 20 schools

The second treatment is a slight modification of the BEEP intervention. Students received a bicycle on the condition that the bicycle be used primarily to travel to school. A Bicycle Supervisory Committee (BSC) was formed consisting mostly of teachers, PTA members or local leaders, and was in charge of monitoring the program. A field mechanic was trained for each school, who provided maintenance checks and repairs for a fee borne by the recipient of the bicycle.

Each school was provided with a startup spare parts kit, but **no contribution** was obtained from the beneficiary students. One of the key advantages of this design is that we are moving one part of the first treatment arm, i.e. *the upfront cost*, which makes the difference between the two-treatment arms cleaner to interpret.

The motivation behind this treatment arm comes from our focus group meetings and the psychology literature. A key take-away from the focus group meetings was that the ownership of the bicycle is more likely to be salient for a girl, when parents do not have to pay any upfront

cost. The academic rationale behind this argument comes from the psychology literature, where the argument is when a good is free, it reduces selfishness due to social norms (Ariely, Gneezy and Haruvy, 2010). In addition, Leon Festinger's (1957) theory on cognitive dissonance and effort justification states that people attach greater value to goods that they have acquired by incurring some effort (in our case, credit constrained households spend money). Thus, if the good is provided for free, it will stick more to the recipient (adolescent girls), and parents would use it less compared to the treatment where they pay an upfront cost of getting the bicycle. The hypothesis is that the "*act of no-payment*" will likely increase the girl's bargaining power when it comes to cycle usage, over and above from the secondary route of increased school attendance.

Control - 55 schools

b. Hypotheses

The intervention is hypothesized to (1) increase mobility for girls through access to a bicycle; (2) improve schooling outcomes (attendance, grade transition) for those that live far away from school and are thus constrained from attending; and (3) raise girls' empowerment. We hypothesize that the mechanisms for the empowerment outcomes happen due to increased mobility, more confidence in school, and through the ownership of a relatively expensive item in an area where girls are not generally given high value items.

In addition, we expect girls in our second treatment arm (no contribution) to have better empowerment measures compared to treatment one. This is because we believe that the ownership of the bicycle is more likely to be salient for a girl when parents do not have to pay any upfront cost. The rationale behind this argument is taken from the psychology literature, which posits that when a good is free, it reduces selfishness due to social norms (Ariely, Gneezy and Haruvy, 2010). In addition, Leon Festinger's (1957) theory on cognitive dissonance and effort justification states that people attach greater value to goods that they have acquired by incurring some effort (in our case, credit constrained households spend money). Thus, if the good is provided for free, it will stick more to the recipient (adolescent girls), and parents would use it less, compared to the treatment scenario where they pay an upfront cost to get the bicycle. The "act of no-payment" will likely increase the girl's bargaining power when it comes to cycle usage, over and above the secondary route of increased school attendance.

c. Study site and population

World Bicycle Relief conducted a needs assessment in several districts in Zambia to identify districts where students walked long distances to school, and where the program was not already implemented. Three districts - Monze, Mazabuka and Kalomo - were ultimately chosen to implement the program. Within the three districts, a total of 100[5] schools were randomly selected from all government schools that met the following criteria:

1. Each school has at least 20 girls enrolled in grades 5, 6 and 7, who walk more than 3 kms to school, and

2. Schools are basic schools, i.e., their starting grade is 1 or lower and their last grade is beyond grade 7 (end of primary) up to grade 9 (last grade before secondary education).

For each school, a Bicycle Supervisory Committee (BSC), consisting of 10-12 members, was formed consisting mostly of teachers, PTA members or local leaders. These committees prepared list of all students in grades 5, 6 and 7, who walked more than 3 kilometers to school. All girls belonging to the list received a bicycle.[6] For the purpose of our study, we collected data on 20-25 randomly selected beneficiary girls from each school, giving us a total sample of 2,466 girls.

d. Power Calculations

To estimate statistical power, we utilized baseline data to determine the intra-cluster correlations (ICCs) for outcome variables. We find that our most critical outcomes have an ICC below 0.10. We then calculate the minimum detectible effect size (MDE) and the size of that effect relative to the control group, expressed as a percentage, assuming an alpha of 0.50 and power of 0.80. The MDE is then calculated using the sample sizes (55 control, 25 treatment one and 20 treatment two schools with 25 girls per school) and the ICC. We expect to be able to identify an MDE of 18% for whether the girl has repeated a grade. For our empowerment measures, we find an MDE of 4 to 8%. We thus expect to be

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[5] Monze (44 schools), Mazabuka (20 schools), Kalomo (36 schools)
[6] If there were multiple girls from the same household, only one bicycle was given to them. A small fraction of the boys also received bicycles, but they are outside the scope of our study.

able to identify relatively small impacts on empowerment with the current design.

e. Timeline and Data Sources

Baseline

The Baseline data was collected during the second term of the school year 2017 (July - August 2017). The girls were asked questions on school attendance, mobility, and measures of bargaining power. They were tested on basic skills in Mathematics and English in addition to a test to measure focus/attention. School characteristics like enrolment and infrastructure were collected from the head teachers.

Randomization

After the collection of the baseline, schools were randomly assigned to one of the two treatment arms or control. 25 schools belonged to the **Payment** arm, 20 belonged to the **No Payment** arm, and the rest 55 schools served as control. Randomization was stratified by district.

Intervention

World Bicycle Relief, in partnership with the Ministry of Education, distributed bicycles to the eligible students in the third term of the school year 2017 (September - October 2017). The distribution took place within the schools and was considered successful by WBR and the research team.

Endline

The Endline data will be collected during the third term of the school year 2018 (September - November 2018), which is about a year after the intervention. The girls will be asked questions on school attendance, mobility, and measures of bargaining power. They will be tested on basic skills in Mathematics and English in addition to a test to measure focus/attention. We also plan to collect non-survey measures to measure girls' bargaining power within the household and their willingness to share an opinion. School characteristics like enrolment and infrastructure will be collected from the head teachers. Basic information on committee demographics and

functioning's will be collected from a representative of the BSC. In addition, administrative data on attendance and end of the year exam performance in grade 7 will be collected from the schools. We detail the exact questions to be asked and how they fit within our various outcomes below.

4. Outcomes

We are interested in understanding the impact of providing a conditional transfer of bicycles to adolescent girls. Our set of outcomes are divided between two categories, primary and secondary, detailed in Table 1. We will conduct multiple hypothesis correction (discussed later) within each of the categories. We also present families of outcomes that will be composed of a variance-weighted index (Anderson, 2008) of the questions listed underneath the bullet. All variables will be first transformed so that a higher value indicates a better or positive outcome. The construction of the index is explained in detail in the Appendix.

In addition to the survey outcomes, we will also use non-survey methods to elicit behavioral decisions. The objective of these activities is to capture real decisions and behavior of the girls in a controlled environment to measure cleanly an outcome that is hard to observe in real life and hard to measure with survey questions. We aim to advance the literature on how one should think about and measure empowerment. In addition to survey measures we are collecting, using non-survey instruments will allow us to observe directly the decision-making, which is often hard to see and will reduce reporting bias in those questions that are susceptible to either social desirability bias or internalized societal norms (see for instance Glennerster, Walsh and Diaz-Martin, 2018). Specifically, we would like to measure two outcomes using non-survey questions: bargaining power at home and willingness to share an opinion.

Bargaining power in the household

This game is played between the girl and the family. The basic idea is that both the girl and the family will rank a list of goods given their preference (example in image 1 in Appendix) independently. Then they will also rank the goods jointly. Then we will select randomly if we give them the good preferred in the independent list (to both the girl and the family) or the one in the join list (also to both the girl and

the family).

We will measure bargaining by comparing where the preferences of the girl lie in the joint list. A girl with more bargaining power will be able to place her preferred good towards the top of the list. This will help us to measure the extent to which the girl can affect the decision made jointly in the household.

Willingness to share an opinion

The girls will perform this activity in groups. The girls will be presented with a moral dilemma, for which there is no right or wrong answer. The girls in the group have to reach a group decision that will be the answer with the highest number of votes.

The girls will be asked to answer their opinion in the dilemma and asked to choose how to submit their answer. They can submit their answer in the "priority" box or in the "non-priority" box. If they select to submit the answer to the "priority" box, the answered will always be read and it will count as a vote. If an agreement has not been reached with the answers in the "priority" box, the answers in the "non-priority" box will be read. These answers will only be read if the answers in the first box were not enough to reach an agreement.

Girls that put the answer in the "priority" box have a stronger willingness to share their opinion than the ones that put it in the "non-priority" box. This will help us to measure the extent to which the girl is willing to share her opinion in a group, as a proxy for future political participation.

The analysis will be done the same as the other outcome variables, comparing treatment and control. We expect the girls that were part of the BEEP program to have more bargaining power at home and more willingness to share an opinion.

5. Estimation Strategy

a. Main effects

For each of the outcomes above, we will examine:

1. Overall impact of the bicycles (T1 and T2 combined, relative to control)

2. Differences between the treatment arms (T1 to T2)

For outcomes in which the same question was asked in both the baseline and the endline survey, our main specification will be:

$$Y_{i,s,t=1} = \beta_0 + \beta_1 T_s + \beta_2 Y_{i,s,t=0} + X'_{i,s}\theta + C'_s\gamma + e_{i,s}$$

Where $Y_{i,s,t=1}$ is the outcome variable of student i in school s measured in post-treatment, $Y_{i,s,t=0}$ is its baseline value, T_s is an indicator for the school assigned to treatment, $X'_{i,s}$ is a vector of individual level controls, C'_s is a vector of school level controls, and $e_{i,s}$ is the error term. Since randomization is at the school level, we will cluster the standard errors at the school level. For outcomes where we do not have baseline values we will not include $Y_{i,s,t=0}$ in the estimation.

β_1 will provide the intent-to-treat effect, which is the effect of being given a bicycle in 2017 on the outcome variable. Since not all those who were given a bicycle will continue to use it, and some of the control group may also have bicycles, we will also present the treatment-on-the-treated effects using an IV estimation.

b. Heterogeneous effects

We will estimate the heterogeneous treatment effects by interacting the treatment status and all control variables with our choice of variable. We are primarily interested in examining the heterogeneous effects with respect to:

1. Baseline time taken to reach school (continuous)

2. Baseline measures of test scores (high versus low dummy)

3. Baseline household wealth index (high versus low dummy)

4. Baseline measure of empowerment index (high versus low dummy)

c. Correction for Multiple Hypotheses Testing

To correct for multiple hypotheses testing, we will use the Benjamini-Hochberg (1995) False Discovery Rate correction. We will show results with the uncorrected and the corrected p-values. Corrections will be made within outcome groupings. So, primary outcomes will all be corrected together, secondary outcomes corrected together, and heterogeneity tests will be corrected together.

d. Tracking and dealing with attrition

Pre-visits will be carried out to all 100 schools prior to the endline. Logistics supervisors will visit schools in advance to:

(i) Notify the school of the team's visit and ensure the sample girls are at school, recording the status (enrolled, transferred, dropped out, etc.) of all sample girls

(ii) Establish what registers the school has/needs to find and note which are available if possible. (The physical capture of these registers will be carried by the field team).

(iii)   Drop off consent forms for the schools, parents and girls.

Pre-visits will allow the research team to confirm their assumptions of expected attrition/drop-out rate. The logistics supervisor will provide the school with a list of names of the girls in the sample and should request that all listed girls attend on the particular day of the visit. At present, pre-visits are estimated to take 12 days with 4 logistics supervisors.

We anticipate that approximately 78% of the full sample or 1,950 respondents will still be enrolled in the school where they were surveyed at baseline. This is based on transition rates collected during the school survey that was carried out at baseline.

Total person-to-person direct interaction with a girl is expected to average 1.5 hours. The surveyors will run multiple groups throughout the day and spend time meeting the teachers, collecting consent forms, checking the registers for sample girls, finding the girls in their classes (who move between lessons), and coordinating the groups, taking pictures of registers, giving snacks to waiting girls etc. The school

day is restricted by specific school hours for different grade classes during the day, which is not consistent across schools. Travel time to/from schools can take up to 2 hours per journey. In doing follow-up tracking we can anticipate that some girls need to be called back to school, some will leave during the day, some will be at home, etc. Back-checkers will visit a randomly selected sub-sample of 10% of the girls, at which point they will document any missing registers, or survey girls who were absent.

To maximize available budgets and minimize bias from tracking, we will utilize a random sampling procedure for those not in school during the follow-up survey. We will randomly sample and then target half of the estimated remaining 22% of the sample (i.e. 275 girls) and attempt to track and survey all of these girls. We anticipate a large proportion of these girls would have transferred or dropped out of school. We will then double weight these selected girls, creating an effective tracking rate of the out-of-school group that will be much higher than the true tracking weight. The number of girls to be tracked will be adjusted dependent on the value of remaining funds.

e. Dealing with outcomes with limited variation

In order to limit noise caused by variables with minimal variation, questions for which 95 percent of observations have the same value within the relevant sample will be omitted from the analysis and will not be included in any indicators or hypothesis tests. In the event that omission decisions result in the exclusion of all constituent variables for an indicator, the indicator will be not be calculated.

Table 1: List of families of outcomes

Outcomes	Domain	Question
Primary	Mobility and Safety	In the past week, how many times did you go outside the house alone to…
		Are you allowed to go alone when…
		I feel safe when…
	School Attendance	Self-reported
		Admin data from registers
	Grade Transition	Dropout
		Grade Progression
	Aspiration	When you finish at school (either end of primary, secondary), what would you like to do?
		How confident are you that you will be able to achieve this?
		If for some reason you cannot (insert answer to previous question), what would you do?
		In two years of time, how confident are you that you will be enrolled in school?
		Do you think you will be working in a job or doing something that makes money in 10 years from now?
		I am going to show you some drawings, could you tell me which ones you think a girl like you can become, if any?
		And from these same drawings, which one would you like to become when you grow up, if any?
		What does your role model do? (Occupation)
		Do you want to do what he/she (Role model) does?
	Locus of Control	Let's say that one day when you are going to school you cannot find a path because of heavy rain or because a tree fell. This is a difficult situation because it is the only way to school and you are already late to school. In situations like this one or other ones similar to this one, you can usually find your way out?
		There are many things that can happen to you in life. Some of them will be good and some will be not so good. For example: - falling over and hurting my knee -forgetting to prepare for an exam or not doing well on an exam - your best friend is upset with you and not talking to you - you were not selected for a school team/club. Do you feel you can control what happens to you in life?
		In general, would you say you are satisfied with your life?
		I feel my life will improve in the future

		What do you think is the ideal age a girl should get married?
		How old would you like to be when you get married?
		Are you currently married, or engaged to someone?
	Fertility and Marriage	If you could choose exactly the number of children to have in your whole life, how many would you like to have?
		Of those children, how many girls and how many boys would you like to have?
		Have you ever been pregnant?
		Do you have any kids?
Secondary Outcomes and Mechanisms	Bicycle ownership and usage	Do you have access to a bicycle that you can use?
		Does this bicycle belong to you?
		In a normal week, how many days do you use a bicycle to go to school?
		Do you use a bicycle during the weekends? How often?
		Do you have to ask permission to use the bicycle?
		How much control do you think you have over the bicycle?
	Time spent traveling to school	Working for the school? (doing school chores like cleaning the classes, etc.)
		Attending extra-curricular activities? (like sport, production unit, club, drama, board games, etc.)
		Studying and doing homework outside of school?
		Helping your family at home or doing other work for them?
		Working to earn money by yourself?
		Being with friends (chatting, playing, games, visiting them at home)?
	Performance - Overall score and fraction in the lowest quartile	Grade 7 end of year exam
		English test
		Mathematics test
		d2 test of attention

	Bargaining	Do you ever have small money of your own (K2 or K5) to use as you would like? This could be money you have earned or that you get from a family member.
		Can you decide on what to spend it on your own?
		Each year there are new fashions (e.g. hair pins) that come out. If you wanted to buy something new and had the money to do so, do you think your parents would allow you?
		Do you own a pair of leggings?
		Do you wear them on their own (if yes)?
		If you don't like what is prepared for dinner, would you tell your mother/guardian you don't like the food or ask them if there is something else to eat?
		When we talked about the activities you perform at home, like (insert activity here). Have you ever skipped doing household chores?
		How often do you say something to your parents if you disagree with what they are saying?
		Do you feel you can to talk to your parents about what you want to be when you grow up?
		Do you think you can talk to your parents if you have problems with friends or at school?
		Do you feel you can talk to your parents about when you wish to get married?
	Self-image	How would you rank yourself academically in your class?
		Compared to your friends, how likely are you to succeed in life?
	Identity	Now let's play again with some drawings. Here you can see six drawings of roles girls usually take in society. Can you put them in order, starting from the one you that describes you better to the one that describes you the least?
		How much do you think you can affect what other people think of your family?
	Pro-sociality	If you notice that one of your friends has a problem, would you help/participate/collaborate?

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Appendix

We will create variance-weighted indices following the methodology proposed by Anderson (2008) (also see Haushofer and Shapiro (2016, QJE) for a recent application).

Anderson (2008) summarizes the index creating process as:

At the most basic level, an index created using this method is a weighted mean of several standardized variables. More weight is assigned to measures that are orthogonal (less similar or less correlated) to other measures. The weights are calculated to maximize the amount of information captured in the index. The index is computed using the following steps.

I.   For all variables, switch signs where necessary so that the positive direction always indicates a "better" outcome.

II.   Create standardized variables ($\tilde{y}$) by demeaning and then by dividing by standard deviation.

III. Compute covariance matrix $\hat{\Sigma}$, which consist of elements:

$$\widehat{\Sigma}_{mn} = \sum_{i=1}^{N_{mn}} \frac{y_{im} - \bar{y}_m}{\sigma_m^y} * \frac{y_{in} - \bar{y}_n}{\sigma_n^y}$$

N_{mn} is the number of observations (total persons with non-missing data for variables m and n),

IV.   Next, we invert the covariance matrix, and define weight w_k for each variable k by summing the entries in the row of the inverted covariance matrix:

$$\hat{\Sigma}^{-1} = \begin{bmatrix} c_{11} & \cdots & c_{1K} \\ \vdots & \ddots & \vdots \\ c_{K1} & \cdots & c_{KK} \end{bmatrix}$$

$$w_k = \sum_{l=1}^{K} c_{kl}$$

V.   Finally create a new variable, $\hat{y}_i$, that is a weighted average of $\tilde{y}_{ik}$ for person i. When constructing $\hat{y}_i$, weight its inputs – standardized variables $\tilde{y}_{ik}$ - by the inverse of the

covariance matrix of the transformed variables. A simple way to do this is to set the weight on each outcome equal to the sum of its row entries in the inverted covariance matrix for area. The index variable is called $\hat{y}_i$ because this transformation yields a generalized least squares estimator (Anderson 2008).

$$\hat{y}_i = \left(\sum_{k \in K} w_k\right)^{-1} \sum_{k \in K_i} w_k * \frac{y_{ik} - \bar{y}_k}{\sigma_k^y}$$