Pre-Analysis Plan:
Salience-Driven Decoy Effects in Stopping Problems

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March 21, 2021

1 Aim of the Study

We develop an experimental design to test for decoy effects in optimal stopping problems, as predicted by salience theory of choice under risk (Bordalo *et al.*, 2012). This experiment complements our existing work (Dertwinkel-Kalt *et al.*, 2020) on salience effects in stopping problems. The (non-parametric) salience predictions on decoy effects that we derive (and plan to test) are inconsistent with other prominent models of choice under risk; in particular, (cumulative) prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992).

2 Salience Theory of Choice under Risk

Consider a choice set $\mathcal{C} = \{X_i\}_{i=1}^n$. The random variables (or *lotteries*) X_1 to X_n are non-negative with a joint cumulative distribution function $F : \mathbb{R}^n_{\geq 0} \to [0, 1]$. A state of the world refers to a tuple of outcomes, $(x_1, \ldots, x_n) \in \mathbb{R}^n_{\geq 0}$. If a random variable is degenerate, we call it a *safe* option.

A *salient thinker* is intrinsically (weakly) risk-averse, but sometimes behaves in a risk-seeking manner, because he inflates the probabilities of the most salient states of the world. More precisely, a salient thinker evaluates monetary outcomes via an increasing and (weakly) concave value function $v : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$, and compares the value of a given lottery, $v(X_i)$, to a *reference point* $R_i = \phi(v(X_1), \ldots, v(X_{i-1}), v(X_{i+1}), \ldots, v(X_n))$. Bordalo *et al.* (2012) assume that the reference point is given by the state-wise average over all alternative options: $R_i = \frac{1}{n-1} \sum_{j \neq i} v(X_j)$. We, in contrast, allow for a more general reference point $\phi : \mathbb{R}^{n-1} \to \mathbb{R}$ that (i) strictly increases in each of its arguments and (ii) satisfies $\phi(z, \ldots, z) = z$. When evaluating a lottery X_i, a salient thinker assigns a subjective probability to the state $(x_1, \ldots, x_n) \in \mathbb{R}^n_{\geq 0}$ that depends on the state's objective probability, and on how salient the realized value $v(x_i)$ is relative to the realized reference point $r_i = \phi(v(x_1), \ldots, v(x_{i-1}), v(x_{i+1}), \ldots, v(x_n))$. Specifically, the salience of a tuple $(v(x_i), r_i) \in \mathbb{R}^2_{\geq 0}$ is measured via a so-called *salience function* that is defined as follows:

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Definition 1 (Salience Function)**.** *A symmetric, bounded, and continuous function* $\sigma : \mathbb{R}^2_{\geq 0} \to \mathbb{R}_{>0}$ *is a* salience function *if and only if it satisfies the following two properties:*[1]

1. Ordering. *Let* $x \geq y$*. Then, for any* $\epsilon, \epsilon' \geq 0$ *with* $\epsilon + \epsilon' > 0$*, we have*

$$\sigma(x + \epsilon, y - \epsilon') > \sigma(x, y).$$

2. Diminishing sensitivity. *For any* $x \neq y$ *and any* $\epsilon > 0$*, we have*

$$\sigma(x + \epsilon, y + \epsilon) < \sigma(x, y).$$

We say that $(v(x_i), r_i) \in \mathbb{R}^2_{\geq 0}$ is the *more salient* the larger its salience value $\sigma(v(x_i), r_i)$ is. Ordering implies that a pair of outcomes is the more salient the more these outcomes differ, thereby capturing the well-known *contrast effect* (e.g., Tversky and Kahneman, 1992; Schkade and Kahneman, 1998). Diminishing sensitivity reflects *Weber's law* of perception and can be understood as a *level effect*: a given contrast in outcomes is more salient at lower outcome levels.

Following Bordalo *et al.* (2012), we assume that a salient thinker chooses an option from the choice set $\mathcal{C}$ in order to maximize his *salience-weighted utility*, which is defined as follows:

Definition 2. *The* salience-weighted utility *of lottery* X_i *evaluated in* $\mathcal{C} = \{X_j\}_{j=1}^n$ *equals*

$$U^s(X|\mathcal{C}) = \frac{1}{\int_{\mathbb{R}^2_{\geq 0}} \sigma\big(v(x_i), r_i\big)\, dF(x_1, \ldots, x_n)} \int_{\mathbb{R}^2_{\geq 0}} v(x_i) \cdot \sigma\big(v(x_i), r_i\big)\, dF(x_1, \ldots, x_n),$$

where $\sigma : \mathbb{R}^2_{\geq 0} \to \mathbb{R}_{>0}$ *is a salience function that is bounded away from zero.*

3 The Stopping Problem

The agent has to decide whether to invest in one of (at most) two assets. In the following, we refer to these assets as X and Y. The assets' prices follow *Arithmetic Brownian Motions* (ABM):

$$dX_t = \mu_X dt + \nu dW_t,$$

and

$$dY_t = \mu_Y dt + \nu dU_t,$$

where both assets share the same initial value $X_0 = z = Y_0$ and the same volatility $\nu \in \mathbb{R}_{>0}$, but Asset X has a larger drift than Asset Y, $\mu_X > \mu_Y$. We further assume that the standard Brownian Motions $(W_t)_{t \in \mathbb{R}_{\geq 0}}$ and $(U_t)_{t \in \mathbb{R}_{\geq 0}}$ are independent of each other.

If the agent invests in either asset, he is restricted to choose a *threshold stopping time*, $\tau_{a,b}$, which is defined as the first leaving time of the interval (a, b) for some $a < z < b$. This means that the agent can sell an asset either at price a or at price b, but at no other prices. We say that a

[1] Bordalo *et al.* (2012) also allow for random variables with negative outcomes and add a third property to ensure that diminishing sensitivity (with respect to zero) reflects to the negative domain: by the *reflection* property, for any $w, x, y, z \geq 0$, it holds that $\sigma(x, y) > \sigma(w, z)$ if and only if $\sigma(-x, -y) > \sigma(-w, -z)$.

threshold stopping time $\tau_{a,b}$ is a *loss-exit strategy* if and only if $b - z > z - a$. Instead of investing, the agent could always decide to receive the initial price of z with certainty. We study two different scenarios in which the agent's choice set is either $\mathcal{C} = \{X_{\tau_{a,b}}, z\}$ or $\mathcal{C} = \{X_{\tau_{a,b}}, Y_{\tau_{a,b}}, z\}$.

Notice that, given $\mu \in \{\mu_X, \mu_Y\}$, the probability of selling an asset at the lower price a is

$$p(\mu) = \frac{\exp(-(2\mu/\nu^2)b) - \exp(-(2\mu/\nu^2)x)}{\exp(-(2\mu/\nu^2)b) - \exp(-(2\mu/\nu^2)a)}. \tag{1}$$

Define $p_X := p(\mu_X)$ and $p_Y := p(\mu_Y)$, and note that $p_X < p_Y$. Since the prices of Asset X and Asset Y evolve independently of each other, the joint distribution of $X_{\tau_{a,b}}$ and $Y_{\tau_{a,b}}$ is as follows:

	$p_X p_Y$	$p_X(1-p_Y)$	$(1-p_X)p_Y$	$(1-p_X)(1-p_Y)$
$X_{\tau_{a,b}}$	a	a	b	b
$Y_{\tau_{a,b}}$	a	b	a	b

Table 1: *Joint distribution of $X_{\tau_{a,b}}$ and $Y_{\tau_{a,b}}$.*

This implies, in particular, that, for the choice set $\mathcal{C} = \{X_{\tau_{a,b}}, Y_{\tau_{a,b}}, z\}$, the reference point R_k relative to which Asset $k \in \{X, Y\}$ is evaluated has the following distribution:

	$p_X p_Y$	$p_X(1-p_Y)$	$(1-p_X)p_Y$	$(1-p_X)(1-p_Y)$
R_X	$\phi\big(v(z), v(a)\big)$	$\phi\big(v(z), v(b)\big)$	$\phi\big(v(z), v(a)\big)$	$\phi\big(v(z), v(b)\big)$
R_Y	$\phi\big(v(z), v(a)\big)$	$\phi\big(v(z), v(a)\big)$	$\phi\big(v(z), v(b)\big)$	$\phi\big(v(z), v(b)\big)$

Table 2: *Distribution of the reference points in the larger choice set.*

4 Salience-Driven Decoy Effects

We derive a few general predictions on how a salient thinker will behave in the two different scenarios. The first result — which is a re-statement of Proposition 3 in Dertwinkel-Kalt *et al.* (2020) — deals with the case of a binary choice set and an Asset X with a non-negative drift. This proposition will serve us as a benchmark in the following: it says that in this case a salient thinker invests in Asset X only if the available stopping time is a loss-exit strategy.

Proposition 1 (Binary Choice Set)**.** *Let $\mathcal{C} = \{X_{\tau_{a,b}}, z\}$, and suppose that the drift of Asset X is non-positive (i.e. $\mu_X \leq 0$). Then, a salient thinker invests in Asset X only if $\tau_{a,b}$ is a loss-exit strategy.*

The second (and main) theoretical result describes a *decoy effect*: if the choice set includes also the dominated Asset Y and if this dominated asset has a sufficiently negative drift, then — compared to the case with a binary choice set — Asset X becomes more attractive to a salient thinker. Hence, a sufficiently "bad" Asset Y serves as a decoy that boosts demand for Asset X.

Proposition 2 (Decoy Effect)**.** *Let $\mathcal{C} = \{X_{\tau_{a,b}}, Y_{\tau_{a,b},}, z\}$, and recall that $\mu_X > \mu_Y$.*

(*a*) *The salient thinker will never invest in the dominated Asset Y.*

(*b*) *The salience-weighted utility derived from investing in Asset X monotonically increases in p_Y.*

(*c*) *There is some $\hat{\mu} \in \mathbb{R} \cup \{-\infty\}$, so that a salient thinker invests in Asset X if and only if $\mu_Y < \hat{\mu}$.*

(*d*) *If the salient thinker invests in Asset X when facing the binary choice set $\{X_{\tau_{a,b}}, z\}$, then $\hat{\mu} \in \mathbb{R}$.*

5 An Experiment on Decoy Effects in Stopping Problems

5.1 Experimental Design and Implementation

We conduct an online experiment in which subjects have to decide whether or not to invest in one (of at most two) assets. The price of any asset (that can be chosen during the experiment) follows an ABM with the same initial price $z = 100$ and the same volatility $\nu = 5$. The drift of the ABM that describes an asset's price can differ, however. There are two types of assets: In each decision, subjects can invest in *Asset Green*, which has a drift $\mu_{Green} = 0$. In some decisions, they can further invest in an *Asset Blue* with a drift $\mu_{Blue} \in \{-10, -20\}$. Subjects can always choose the outside option of *No Investment*, in which case they receive an asset's initial price of $z = 100$ Taler (an experimental currency that is converted into Pounds at a ratio of 60:1) with certainty. Figure 1 illustrates the decision screens with only one (left panel) or two (right panel) assets.

Figure 1: *Screenshots of the decision screen with and without a decoy.*

If a subject decides to invest in an asset, she can sell it at pre-specified prices 90 and 190. More precisely, if a subject invests, then the price of the asset will change until it reaches either 90 or 190. The asset cannot be sold at other prices. Notice that, motivated by Proposition 1, this "selling strategy" is a loss-exit strategy and, thus, potentially attractive to a salient thinker.

Each subject makes three investment decisions: one decision with a binary choice set (*Asset Green* vs. *No Investment*) and two with a larger choice set (*Asset Green* vs. *Asset Blue* vs. *No Investment*) for $\mu_{Blue} \in \{-10, -20\}$. The order of decisions is randomized at the subject level.

To explain the drift of an ABM to the subjects, they have to draw three sample paths from the underlying process and, in addition, they see an overview of five additional sample paths of this process before making a decision (see Figure 2 for examples of the latter with and without a decoy). The sample paths are randomly drawn at the subject level; that is, different subjects see different sample paths of the same underlying process.

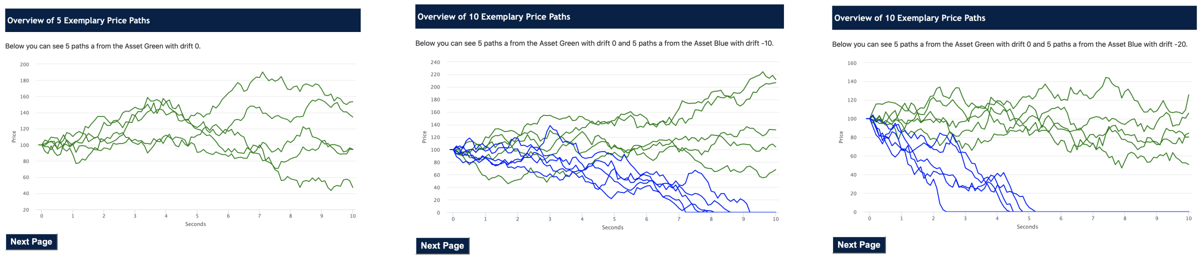

Figure 2: *Screenshots of the sampling screens with and without a decoy.*

At the end of the experiment, one of the three investment decisions will be randomly drawn by the computer to be payoff-relevant. All subjects receive an additional £3 for their participation in the experiment.

We plan to run the experiment online via the Oxford lab with a total number of $n = 250$ subjects. The experiment will take place in March/April 2021.

5.2 Hypotheses and Statistical Tests

Our main outcome of interest is the share of subjects investing in *Asset Green*. Let $\omega_{i,k}$ be a binary indicator that takes a value of 1 if subject $i \in \{1, \dots, n\}$ invests in *Asset Green* in decision $k \in \{1, 2, 3\}$ and a value of zero otherwise. We sort the decisions in the following way: $k = 1$ refers to the binary choice set (*no decoy*); $k = 2$ refers to the decision where *Asset Blue* has a drift of $\mu_{Blue} = -10$; and $k = 3$ refers to the decision where *Asset Blue* has a drift of $\mu_{Blue} = -20$.

Before we can state our hypotheses, we need to make a few assumptions on how to deal with subjects choosing the dominated *Asset Blue*. We assume that (1) any subject choosing the dominated *Asset Blue* does so by mistake, and (2) the probability of making a mistake is independent of whether a subject would have chosen *Asset Green* or *No Investment* otherwise. Under these two assumptions, we can simply drop the choices in favor of the dominated asset (without creating any bias), and define our outcome variables as follows: denote as $\mathcal{I}_k \subseteq \{1, \dots, n\}$ the set of subjects who do *not* choose the dominated asset in decision $k \in \{2, 3\}$, and define

$$\overline{\omega}_{no} := \frac{1}{n} \sum_{i=1}^{n} \omega_{i,1}, \quad \text{and} \quad \overline{\omega}_{10} := \frac{1}{|\mathcal{I}_2|} \sum_{i \in \mathcal{I}_2} \omega_{i,2}, \quad \text{and} \quad \overline{\omega}_{20} := \frac{1}{|\mathcal{I}_3|} \sum_{i \in \mathcal{I}_3} \omega_{i,3}.$$

We then hypothesize:

Hypothesis 1. $\overline{\omega}_{20} \geq \overline{\omega}_{10}$.

Hypothesis 2. $\overline{\omega}_{10} \neq \overline{\omega}_{no}$.

Hypothesis 3. $\overline{\omega}_{20} \neq \overline{\omega}_{no}$.

The first hypothesis follows from Proposition 2 (b), and provides a clear test of the salience model that we have set up. Also the second and the third hypothesis are consistent with salience theory, but neither allows us to falsify the theory (see Proposition 2). Notably, it could be the case — and salience theory allows for this — that subjects completely neglect "too bad" decoys: in this case, Hypothesis 1 is not implied by salience theory.

To test for Hypothesis 1, we conduct a (one sided) t-test of the null-hypothesis $\overline{\omega}_{20} < \overline{\omega}_{10}$, with standard errors being clustered at the subject level. To test for Hypothesis 2, we conduct a (two sided) t-test of the null-hypothesis $\overline{\omega}_{10} = \overline{\omega}_{no}$, with standard errors being clustered at the subject level. To test for Hypothesis 3, we conduct a (two sided) t-test of the null-hypothesis $\overline{\omega}_{20} = \overline{\omega}_{no}$, with standard errors being clustered at the subject level.

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Appendix A: Proofs

Proof of Proposition 1. See Proposition 3 in Dertwinkel-Kalt *et al.* (2020). □

Proof of Proposition 2. Part (b). The salience-weighted utility from investing in Asset X is

$$U^s(X_{\tau_{a,b}}|\mathcal{C}) = v(a)\pi(p_X, p_Y) + v(b)\big[1 - \pi(p_X, p_Y)\big],$$

where

$$\pi(p_X, p_Y) := \frac{p_X s_a(p_Y)}{p_X s_a(p_Y) + (1 - p_X)s_b(p_Y)},$$

with $s_a(p_Y) := p_Y\sigma\big(v(a), \phi(v(z), v(a))\big) + (1 - p_Y)\sigma\big(v(a), \phi(v(z), v(b))\big)$ being the average salience of a and $s_b(p_Y) := p_Y\sigma\big(v(b), \phi(v(z), v(a))\big) + (1 - p_Y)\sigma\big(v(b), \phi(v(z), v(b))\big)$ being that of b.

Since $\phi(v(z), v(b)) > \phi(v(z), v(a))$ and, thus, $\sigma\big(v(a), \phi(v(z), v(b))\big) > \sigma\big(v(a), \phi(v(z), v(a))\big)$ by the ordering property, $s_a(p_Y)$ is strictly decreasing in p_Y. Analogously, ordering implies that $\sigma\big(v(b), \phi(v(z), v(a))\big) > \sigma\big(v(b), \phi(v(z), v(b))\big)$, so that $s_b(p_Y)$ is strictly increasing in p_Y. It follows that $\pi(p_X, p_Y)$ is strictly decreasing and, thus, $U^s(X_{\tau_{a,b}}|\mathcal{C})$ is strictly increasing in p_Y.

Part (a). Next, we observe that

$$\begin{aligned}\frac{\partial}{\partial p_X}\pi(p_X, p_Y) &= \frac{s_a(p_Y)\big[p_X s_a(p_Y) + (1 - p_X)s_b(p_Y)\big] - p_X s_a(p_Y)\big[s_a(p_Y) - s_b(p_Y)\big]}{\big(p_X s_a(p_Y) + (1 - p_X)s_b(p_Y)\big)^2} \\ &= \frac{s_a(p_Y)s_b(p_Y)}{\big(p_X s_a(p_Y) + (1 - p_X)s_b(p_Y)\big)^2} > 0,\end{aligned}$$

which, in turn, implies that $U^s(X_{\tau_{a,b}}|\mathcal{C})$ is strictly decreasing in p_X.

Combining this with Part (b) and the fact that $p_Y > p_X$, we conclude:

$$\begin{aligned}U^s(X_{\tau_{a,b}}|\mathcal{C}) &= v(a)\pi(p_X, p_Y) + v(b)\big[1 - \pi(p_X, p_Y)\big] \\ &> v(a)\pi(p_X, p_X) + v(b)\big[1 - \pi(p_X, p_X)\big] \\ &> v(a)\pi(p_Y, p_X) + v(b)\big[1 - \pi(p_Y, p_X)\big] = U^s(Y_{\tau_{a,b}}|\mathcal{C}).\end{aligned}$$

Part (c). Follows immediately from the fact that $U^s(X_{\tau_{a,b}}|\mathcal{C})$ is increasing in p_Y.

Part (d). Given the binary choice set $\mathcal{C}' = \{X_{\tau_{a,b}}, z\}$, the salience-weighted utility from investing in Asset X is given by $U^s(X_{\tau_{a,b}}|\mathcal{C}') = v(a)\tilde{\pi} + v(b)\big[1 - \tilde{\pi}\big]$, where

$$\tilde{\pi} := \frac{p_X\sigma\big(v(a), v(z)\big)}{p_X\sigma\big(v(a), v(z)\big) + (1 - p_X)\sigma\big(v(b), v(z)\big)}.$$

Notice that

$$\lim_{p_Y \to 1}\pi(p_X, p_Y) = \frac{p_X\sigma\big(v(a), \phi(v(z), v(a))\big)}{p_X\sigma\big(v(a), \phi(v(z), v(a))\big) + (1 - p_X)\sigma\big(v(b), \phi(v(z), v(a))\big)} < \tilde{\pi},$$

since, by ordering, $\sigma\big(v(a), \phi(v(z), v(a))\big) < \sigma\big(v(a), v(z)\big)$ and $\sigma\big(v(b), \phi(v(z), v(a))\big) > \sigma\big(v(b), v(z)\big)$. Hence, if $U^s(X_{\tau_{a,b}}|\mathcal{C}') > 0$, then also $\lim_{p_Y \to 1} U^s(X_{\tau_{a,b}}|\mathcal{C}) > 0$. □